The roots of Granular Computing

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Background

“ … Granular Computing – a general computation theory for effectively using granules […] to build an efficient computational model for complex applications …”

IEEE-GrC’2006 conference information

Background

- **L.A. Zadeh**, "Toward a theory of fuzzy information granulation and its centrality in human reasoning and fuzzy logic", *Fuzy Sets and Systems,* 90, 111-127, 1997
- **T.Y.Lin**, "Granular computing on binary relations", in L.Polkowski and A.Skowron (Eds.) *Rough sets in knowledge discovery,* Physica-Verlag, Heildelberg, 286-318, 1998

Background

Given the broad cross-disciplinary setting in which Granular Computing arose it is natural to try to clarify the distinctiveness of GC from the underpinning constituent disciplines and other computational paradigms.

Components of GC

"… perceptions (to provide source data) and computation (to extract knowledge from data) …"

L. Zadeh [1965, 1972, 1983, 2006]

Components of GC

"… structured thinking at the philosophical level and structured problem solving at the practical level …"

YY.Yao, [2002, 2004, 2005]

Components of GC

"… knowledge (empirical knowledge + commonsense knowledge + regular knowledge) and processing formalism (neural network + sensor motor + expert system) …"

Y. Zhong, [2002, 2006]

Re-visiting the origins

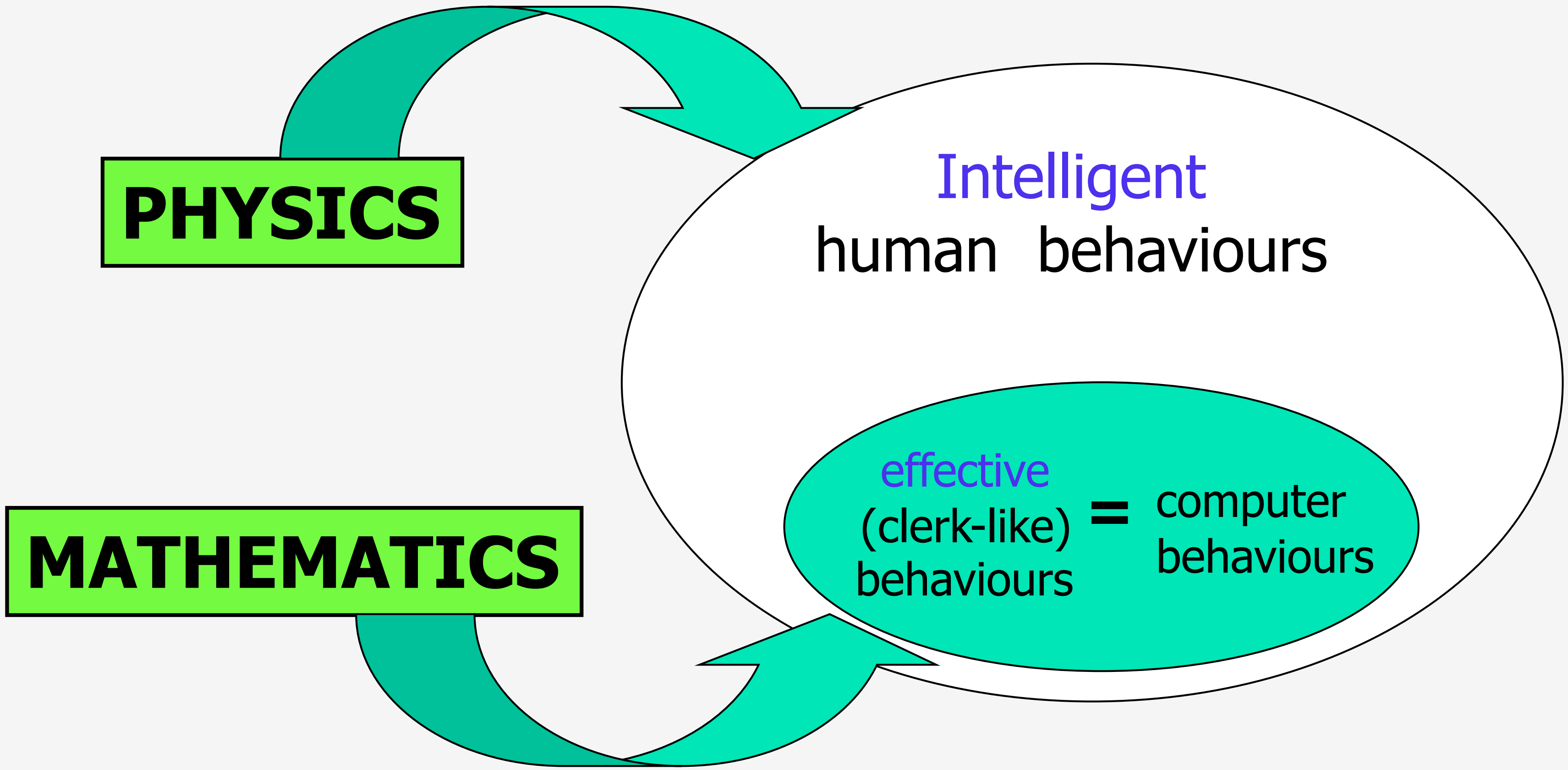
" … fuzzy information granulation in an intuitive form underlies human problem solving. […] How this could be done (by machines) is a challenge that has not as yet been met ..."

L.Zadeh, [1997]

Characteristics of behaviours

- Effective
  - Processing of countable set of symbols *(information)*
  - Operation to "fixed rule"
  - Operation without insight
  - Operation without understanding
- Intelligent
  - Processing of uncountable abstractions *(knowledge)*
  - Keeping "open mind" about operation
  - Essential insight
  - Essential understanding

Formal frameworks for study of intelligence



Effective = clerical labour, working to fixed rule, and without understanding

processing of symbols (information)

Re-visiting the origins

<u>Human problem solving</u>

➔ overcomes limitations of the UTM

- countable set of symbols;
- countable set of states;
- no insight/intuition;

Turing, [1934]

Re-visiting the origins

Human problem solving

- What symbols humans process?
- How these symbols are generated?
- Are these symbols countable?
- Do symbols externalise insight?

Why granulation?

Definition of granulation

Grouping of elements based on their indistinguishability, similarity, proximity or functionality

The emphasis here is on constructive generation of granules which in itself does not distinguish granulation from clustering

Why granulation?

Revised definition of granulation

Semantically meaningful grouping of elements based on their indistinguishability, similarity, proximity or functionality

The emphasis here is on generation of symbols that are used in human problem solving.
The symbols satisfy properties of similarity etc. but also have a meaning associated with them.

Semantics of granules

Is the semantics of granules derived from the granulated objects?

Can we interpret granulation as a mere formation of subsets within the “intuitive set theory” proposed by Cantor?

Granulation and set theory

If the granules are interpreted uniformly as sets/subsets it is inevitable that one also arrives at the paradoxes of set theory in the context of granulation.

"cardinality of set of all sets" (Cantor)
"definition of a set that is not a member of itself" (Russel)

Lessons from set theory

Zermelo-Frankel set theory –
prevents inconsistencies through the introduction of axiom schemas (implying the need for infinitely many axioms)

Lessons from set theory

Von Neumann-Bernays-Goedel (NBG) set theory
- prevents inconsistencies through the introduction of the concept of class in addition to the concept of set.

Lessons from set theory

- Membership relation $a \in b$ is only defined if *a* is a set and *b* is either a set or a class
- Granulation process transforms the semantics of the granulated entities
- The operation on classes in NBG is consistent with operations on sets in intuitive sets theory
- A set represents a class if every element of the set is an element of the class
- There are classes that do not have representations (eg. class of all sets that do not contain themselves)
- NBG provides a framework for discussing hierarchy of different granulations without the risk of inconsistency

Lessons from set theory

Set theory of types

A linear hierarchy of types: with type *0* consisting of objects of undecided type and, for each natural number *n*, type *n+1* objects are sets of type *n* objects

The conclusions that can be drawn from this framework with respect of granulation are exactly the same as that drawn from the NBG.

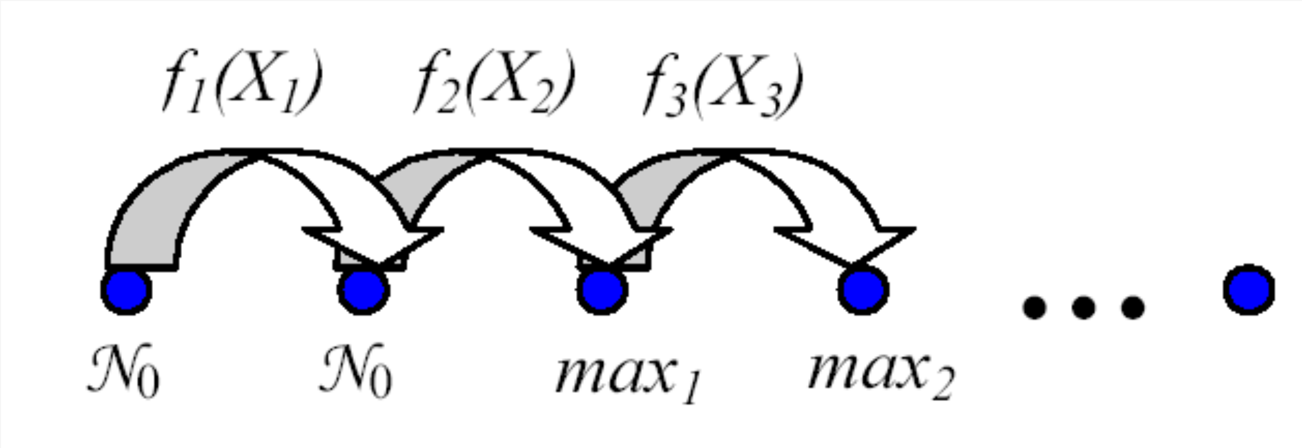
Conclusions from set theory

- The concept of granulation is necessary to denote the <u>semanticaly meaningful grouping</u> entities
- Granulation interpreted through set theory is very different from clustering
- Set theoretical interpretation of granulation enables consistent representation of a hierarchy of information granules

Semantics of granules

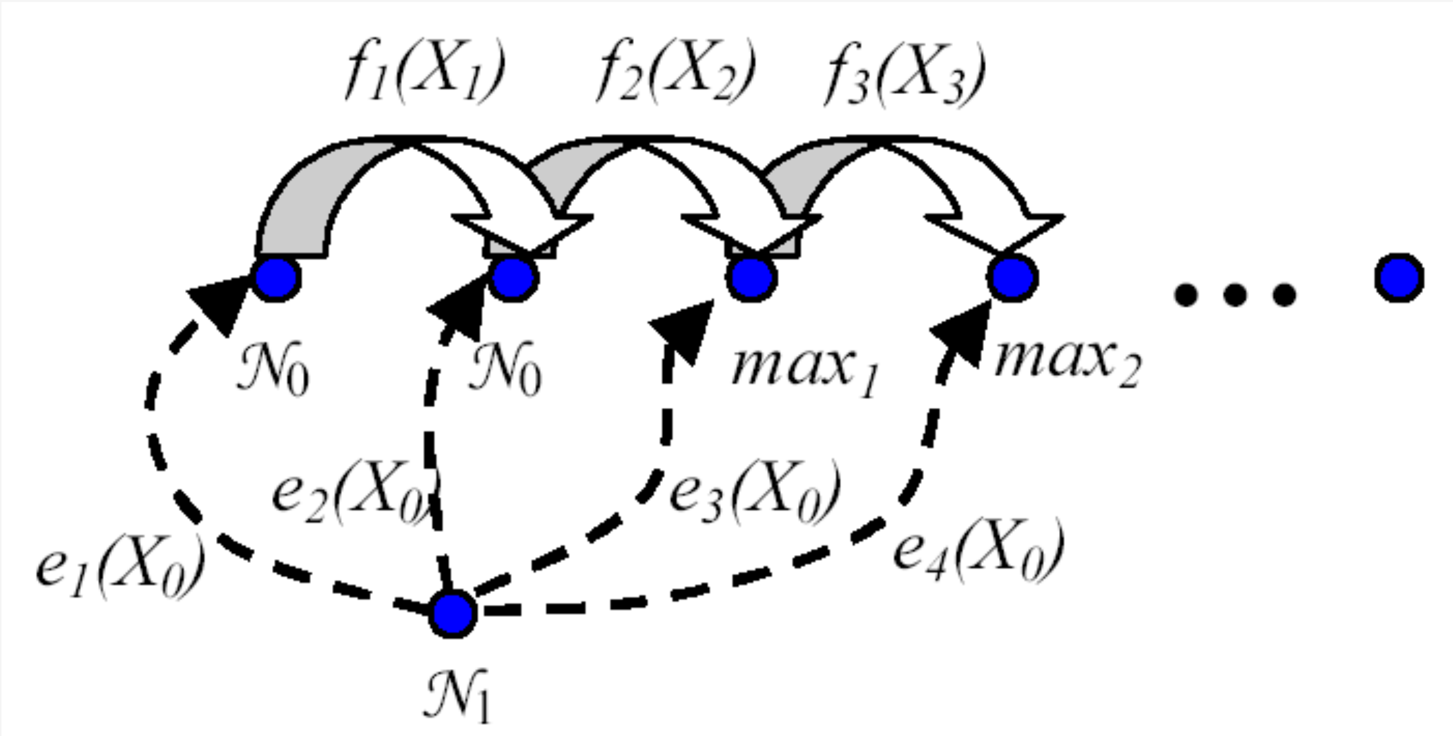
- How is the meaning (semantics) instilled into information granules?
- Is the meaning instilled through algorithmic processing or is it a process that is independent of computation?

Hierarchy of clustering



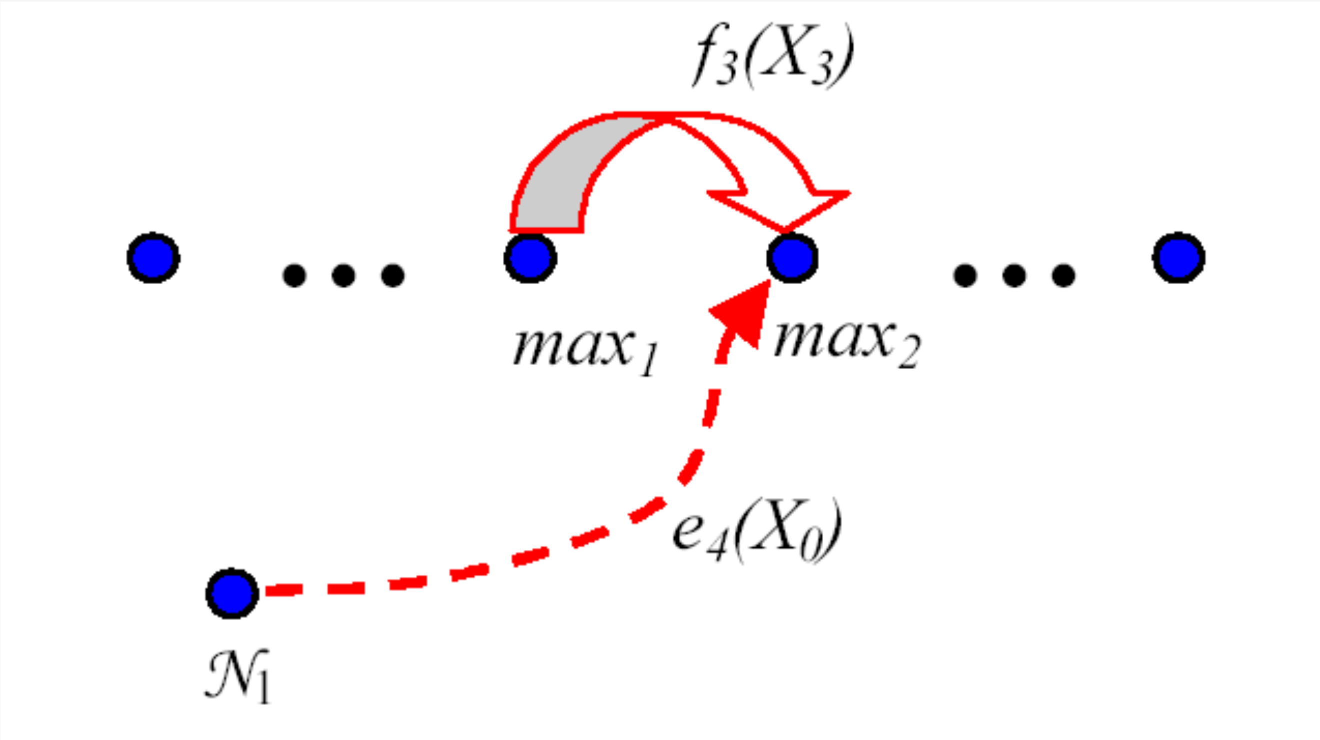
UTM computable mappings:
- Infinite (countable) input set onto infinite (countable) output set
- Infinite (countable) input set onto finite output set;
- Finite input set onto finite output set.

Hierarchy of granulation



Granules verified against "real life" ➔ experimentation function

Granular Computing



New definition

We propose that Granular Computing is defined as a structured combination of algorithmic abstraction of data and non-algorithmic (empirical) evaluation of the semantics of these abstractions.

Consequences/conclusions

- Emphasises the complementarity of the two constituent mappings within GC;
- Justifies hyper-computational nature GC;
- Places physics and set theory as the theoretical foundation of GC;
- Helps to avoid confusion between GC and purely algorithmic data processing while taking full advantage of the advances in the latter;