A Construction Approach for Examination Timetabling based on Adaptive Decomposition and Ordering

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In this study, we investigate an adaptive decomposition strategy that automatically divides examinations into *difficult* and *easy* sets in constructing an examination timetable. The examinations in the difficult set are considered to be hard to place and hence are listed before the ones in the easy set. Moreover, examinations within each set are ordered using different strategies based on the graph colouring heuristics. Initially, all the examinations are placed into the easy set. During the construction process, the examinations that cannot be scheduled are identified as the ones causing infeasibility and are moved forward in the difficult set in order that they could be scheduled earlier than the others during subsequent timetable construction attempts. The examinations that can be scheduled initially remain in the easy set. The proposed approach which incorporates different ordering and shuffling strategies is tested on Carter benchmark problems and the empirical results show that its performance is comparable to other constructive approaches.

Keywords: examination timetabling, decomposition, adaptive, graph colouring heuristic, roulette wheel selection.

1 Introduction

Timetabling attracts numerous researchers and practitioners due to its challenging nature. Timetabling problems are NP hard real-world problems (Even et al. 1976) that are hard to solve and often require considerable amount of wither human or computational time or both. There are many types of timetabling problems e.g. university course timetabling, nurse rostering, etc. The focus of this study is the examination timetabling problem. Principally, the examination timetabling problem is concerned with the scheduling of a list of examinations into a restricted number of time-slots while satisfying a defined set of constraints. Hard constraints must be satisfied in creating a feasible solution e.g.. no student should take two examinations at the same time. Soft constraints on the other hand can be broken but it is desirable to satisfy them as much as possible. The evaluation of the degree these soft constraints are satisfied provides an indication of the overall quality of a given solution. In relation to examination timetabling, evaluating the average cost of student spread in the timetable as an indicator of how 'good' a given solution is was introduced by Carter et al. (1996). More overview on examination timetabling problem and the constraints can be found in (Carter and Laporte 1996; Carter et al. 1996; Petrovic and Burke 2004; Qu et al. 2009).

Considering that the only constraint dealt with is the one that requires no student sitting two examinations at the same time, formulating the examination timetabling problem in this way allows close similarity to be made with graph colouring problems. Ülker et al. (2007) discusses a grouping representation for this type of examination timetabling problems. The vertices and edges of a graph denote the examinations and the conflicting examinations that should not be scheduled at the same time, respectively, where the colour of a vertex denotes a time-slot in the timetable. The heuristic ordering methods for graph colouring are considered constructive approaches. These approaches have been used to find an initial solution before the improvement phase. There are several heuristic ordering methods in examination timetabling i.e. largest degree, saturation degree, largest weighted degree, largest enrolment and colour degree (Carter 1986; Carter and Laporte 1996, Burke et al. 2004a).

A wide variety of approaches has been applied to examination timetabling. The cluster-based/decomposition approaches group the examinations into smaller sub-groups in order to reduce the complexity of the problem. Examples of such approaches can be found in Table 1. Examination timetabling also can be presented in the framework of exact method or approximation algorithm. The constraint based method is one of the examples that involved a lot of constraints especially when the problem size is increasing. The constraint based method can be exemplified as constraint logic programming and constraint satisfaction problems (Carter and Laporte 1996).

Much research in the area of timetabling has utilized meta-heuristic approaches with great success. Initially, the meta-heuristic approach incorporates one or more initial solution at the beginning. Their employment has been focused to avoid the run getting stuck in local optima and allowing the search to cover more area within the overall search space (Petrovic and Burke 2004). The meta-heuristic technique applied can be divided into two category i.e. single-point based search methodology that deal with a single candidate solution at each iteration and multi-point based search methodology that employ a population of candidate solutions during the search process. Example of single-point based search methodology includes variable neighbourhood search, large neighbourhood search, iterated local search, greedy randomised adaptive search procedure (GRASP), tabu search, simulated annealing and great deluge algorithm. Genetic algorithms, memetic algorithms and ant algorithms are categorised into multi-point based search methodology.

Recent trends in search methodology applied to timetabling attempts to works at a higher level of generality than meta-heuristisc. These include hyper-heuristic which are methodologies that perform search in heuristics space (Burke et al. 2003; Özcan et al. 2008) and case-based reasoning approaches. Other recent approaches to timetabling are fuzzy approaches and neural network**.** A summary of various methodologies for examination timetabling problem is provided in Table 1.

Recent study in timetabling has focused on constructive approaches in obtaining high solution quality. Graph colouring heuristics have been customized with adaptive approach to order the examination based on their difficulty (Burke and Newall 2004). This study utilised the framework of 'squeaky wheel optimisation' approach (Joslin and Clement 1999) where the difficulty of an examination is identified based on its infeasibility in a previous iteration. The difficulty indicator of an examination was increased based on a certain parameter so that it could be scheduled earlier in the next iteration. In 2009, Abdul Rahman et al. extended this study by introducing more strategies to choose examination and time-slots during an iteration. In another adaptive approach, Casey and Thompson (2003) developed a GRASP algorithm for solving examination timetabling problems. In the approach, the next examination to be scheduled from the heuristic orderings is chosen from the top items in the list (called candidate list) using roulette wheel selection and then the chosen examination is assigned to the first available slot.

Table 1. Some representative studies for solving examination timetabling problems

Methodology	**Reference(s)**
Cluster-based/decomposition	Balakrishnan et al. (1992), Burke and Newall (1999), Qu and Burke (2007)
Tabu search	Di Gaspero and Schaerf (2001), White and Xie (2001)
Simulated annealing	Thompson and Downsland (1998), Merlot et al. (2003)
Great deluge algorithm	Burke et al. (2004b)
Variable neighbourhood search	Burke et al. (2010)
Large neighbourhood search	Abdullah et al. (2007)
Iterated local search	Caramia et al. (2001)
GRASP	Casey and Thompson (2003)
Genetic algorithms	Burke et al. (1995), Ülker et al. (2007)
Memetic algorithms	Burke and Newall (1999), Ozcan and Ersoy (2005), Ersoy et al. (2007)
Ant algorithms	Eley (2007)
Exact method	Boizumault et al. (1996), David (1998), Merlot et al. (2003)
Multi-objective	Petrovic & Bykov (2003), Ülker et al. (2007)
Hyper-heuristic	Bilgin et al. (2007), Ersoy et al. (2007), Pillay and Banzhaf (2009)
Case-based reasoning	Burke et al. (2006)
Fuzzy approaches	Asmuni et al. (2009)

Neural network	Corr et al. (2006)
Constructive approaches	Burke and Newall (2004), Abdul Rahman et al. (2009)

The study of constructive approaches within a decomposition strategy is very rare in timetabling area. In decomposition approaches, a problem is divided into a smaller sub-problems and the solution is obtained from each sub-problem considering the related constraints. Even though this strategy could reduce the processing time it could affect adversely the quality of the final solution because of the drastic reduction of the search space during the scheduling process (Carter and Laporte 1996). The sub-problems need to be carefully reunited to sustain the optimality of the solution.

The study by Qu and Burke (2007) describes an adaptive decomposition approach for constructing an examination timetable. This paper draws upon the research on similar adaptive approaches that make use of a decomposition strategy. We propose an approach which divides the problem into two sub-problems. We adopt the same naming convention introduced by Qu and Burke (2007) for these sets as *difficult* and *easy*. In this study, the problem is decomposed into difficult and easy sets during each iteration. A timetable is constructed based on the associated heuristic ordering for each set. We also introduce an additional set of examinations which is located in between the difficult and easy sets, which is referred to as the *boundary* set. This study describes several mechanisms associated with the boundary set in order to vary the search space of solutions. In Section 2, we present the details of our approach based on adaptive decomposition and ordering for examination timetabling. Section 3 describes the experimental data and discusses the results. Finally, the conclusion is provided in Section 4.

2 Automated Decomposition and Ordering of Examinations

Most of the timetabling approaches do not make use of the information obtained from the process of building an infeasible timetable. The examinations causing the infeasibility of a solution provide an indication that those examinations are very difficult to place and should perhaps be treated in different ways. We propose a general constructive framework as presented in Pseudocode 1 for solving the examination timetabling problems based on the automated decomposition of a set of examinations into two sets i.e. difficult and easy. At each iteration, a new solution is constructed from an ordered list of examinations. The difficult set consists of the examinations that cannot be placed into a time-slot within the timetable due to some conflicts with the other examinations from the previous iteration. These examinations need to be associated with a large penalty imposed on the unplaced examinations. On the other hand, the examinations in the easy set cause no violations during the timetabling. In our approach, all the examinations that contribute to the infeasibility in a solution are given priority. They are moved forward in the ordered list of examinations and treated first. Such examinations are detected and included in the difficult set at each iteration and a predefined ordering strategy is employed before their successive assignment to the available timeslots. The remaining examinations that generate no feasibility issues are placed into the easy set and the original ordering of those examinations is maintained. In order to incorporate a stochastic component for the selection of examinations from the generated ordering, some shuffling strategies are utilised. The following subsections discuss these strategies.

Pseudocode 1: Improvement and construction of a timetable based on automated decomposition and ordering of examinations.

```
E={e1, e2,…., eN}
BoundarySize=δ
FOR i=0 to MAXIter
    EasySet=E; DifficultSet=ø; TempSet=ø
    OrderExams(EasySet, DifficultSet)
    FOR each exam e in order of chosen heuristic
        IF e can be schedule in the timetable THEN
            TempSet=TempSet∪{e}
            Schedule e in the time-slot of least penalty
            IF more than one time-slots has least penalty
                Choose one time-slot randomly
            ENDIF
            ELSE
               Move forward exam e
        ELSE
            EasySet=TempSet;
            DifficultSet=E-EasySet
        ENDFOR
    ENDFOR
    ProcessAllSets()
    Evaluate solution and store if best found so far
ENDFOR
```

2.1 Interaction between Difficult and Easy Sets through a Boundary Set

An adaptive decomposition approach is experimented with two graph colouring heuristics for generating the initial ordering of examinations. We have tested the largest degree heuristic that orders the examinations decreasingly with respect to the number of conflicts with each examination and the saturation degree heuristic that dynamically orders the unscheduled examinations based on the number of available time-slots for each of them during the timetable construction. The reason for testing these two graph colouring heuristics is to compare their achievement in terms of solution quality and the contribution of difficult set size, as they represent static and dynamic ordering heuristics. Initially, all the examinations are considered to be a member of the easy set (as illustrated in Figure 1(a)).

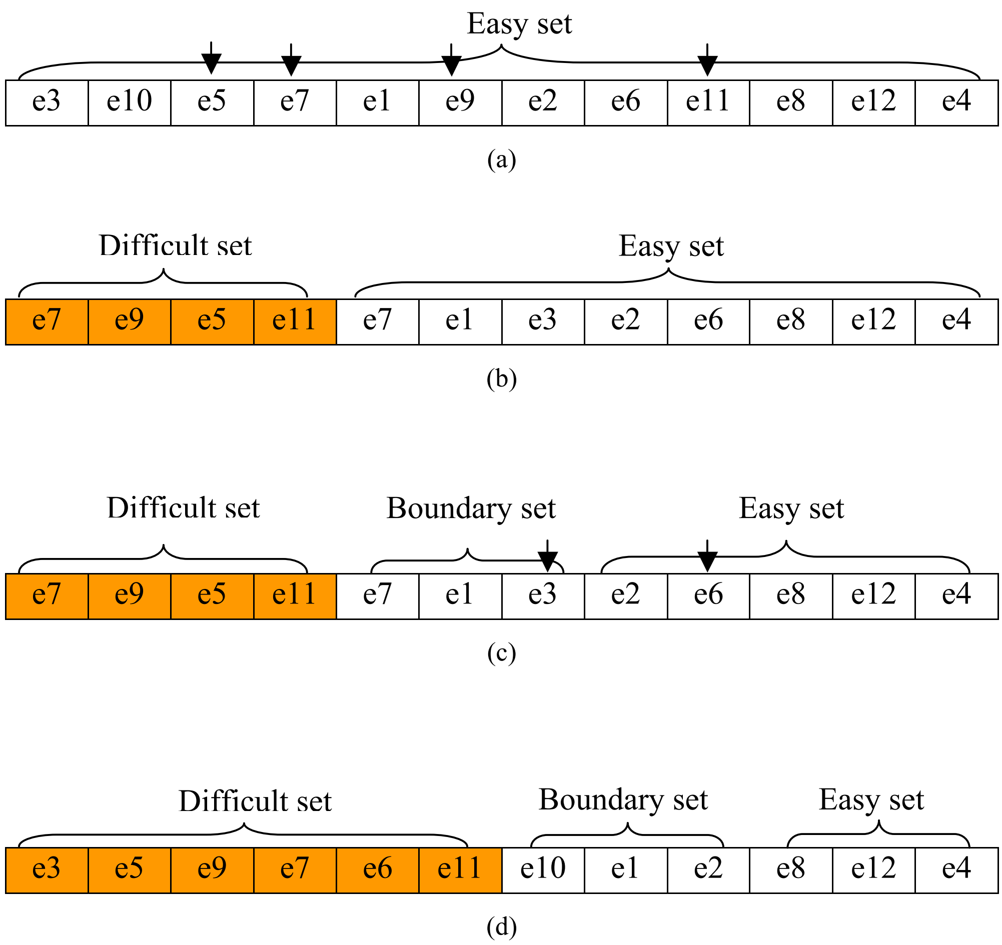


Figure 1. (a) All examinations are in a easy set in the first iteration and examinations that cause infeasibility are marked, (b) difficult and easy sets after an iteration resulting with an infeasible solution, (c) boundary set with a prefixed size is added to the difficult set after an iteration and reordering is performed, (d) the step in (a) is repeated and the infeasible examinations are placed in the difficult size, the size of difficult set increased.

During an iteration, the examinations causing infeasibility are identified. As in Figure 1(a), all such examinations are marked as a member of the difficult set to be moved forward towards the top of the list of examinations (Figure 1(b)), while the examinations that caused no violation during the assignment to a time-slot remain in the easy set. In Figure 1(c), the boundary set is created between the difficult and easy set and is merged with the difficult set before a reordering is performed to the difficult set. In the next iteration, more infeasible examinations are detected and included in the difficult set. Consequently, the size of the difficult set is increased from one iteration to another.

2.2 Swapping the Examinations Between Difficult and Boundary Sets

In this strategy, an additional stage is introduced in Pseudocode 1, ProcessAllSets. The difficult set and the boundary set are shuffled by swapping the examinations in between them randomly. Occasionally, the examination causing infeasibility is not necessarily the one that is very difficult to schedule. The infeasibility may happen due to the previous assignment and ordering. This strategy introduces the opportunity for some of the examinations in the difficult set to be chosen later in the timetable. There is also a possibility that the examinations in the boundary set are swapped back to the original set because this process is done randomly. Figure 2 illustrates how the swapping of examinations between two sets might take place.

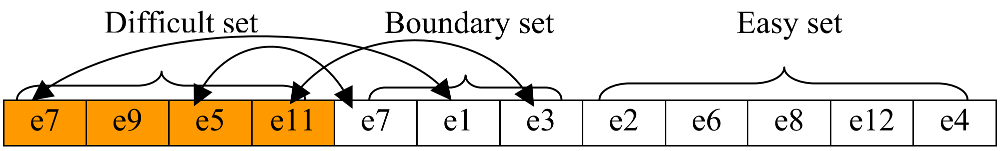


Figure 2: The boundary set is swapped with the difficult set and is reordered before assigning examinations to the time-slots.

2.3 Roulette Wheel Selection for Examinations

We utilised a roulette wheel selection strategy that incorporates a stochastic element in choosing examinations before assigning them to the time-slots. If there is no improvement to the algorithm for a certain time, a list of examination of size n was chosen from the ordered list in the difficult set and an examination is chosen based on probability. The probabilities of an examination being chosen were calculated based on a score, s_i of each examination in the list of size n. The new size of the difficult set will be the set which includes the size of boundary whenever there is improvement to the solution quality. The score value, s_i is a dynamic measure that is obtained from the largest and saturation degree values (as in equation 1), where Num_clash_i is the number of examinations in conflict with the examination i, Max_clash is the maximum number of conflicts with all examinations, Sat_degree_i is the saturation degree value for the examination i and *Num_slots* is the number of time-slots given to the specified problem. *Sat_degree* value in this problem is initialised as 1.

$$s_i = \frac{Num\_clash_i}{Max\_clash} + \frac{Sat\_degree_i}{Num\_slots} \qquad (1)$$

The probability, p_i of an examination being chosen from n list of examinations is,

$$p_i = \frac{s_i}{\sum_{i=0}^{n-1} s_i}, \qquad (2)$$

A random number from (0, 1) is obtained in order to choose an examination from a list of examination of size n. An examination with higher score value will have greater chance to be chosen.

2.4 Comparison of Our Approach to a Previous Study

Qu and Burke (2007) previously proposed an adaptive decomposition approach to construct examination timetables. Their approach starts with an initial ordering of examinations using a graph colouring heuristic, namely saturation degree. In the approach, a perturbation is made by randomly swapping two examinations in order to obtain a *better* ordering. Examinations are then decomposed into two sets: difficult and easy.

The initial size of the difficult and easy sets are prefixed as half of the number of examinations in a given problem as shown in Figure 3(a). At each iteration, the size of the difficult set is modified according to the feasibility of the solution. If the solution is infeasible after the adjustment of the ordering of examinations then the first examination that causes infeasibility (e.g. e11) is moved forward for a fixed number of places (e.g., five as illustrated in Figure 3(b)). The size of the difficult set is then re-set to the point where the difficult examination is placed. Otherwise, if feasible solution or an improved solution is obtained, the size of the difficult set is increased (Figure 3(c)).

Our approach initialises with the easy set including all the examinations and the difficult set is formed during each construction phase at each iteration. The size of the difficult set depends on the number of unscheduled examinations that cannot be assigned to any time-slot from all previous iterations. The size of the difficult set never decreases and after a certain iteration, the number of examinations in the difficult set might be sustained. On the other hand, in the previous approach, the size of the difficult set is prefixed and increased when the feasible solution or improved solution is obtained statically. The set is allowed to shrink as well. Additionally, the previously proposed approach uses an initial ordering and reorders all the examinations without using a heuristic, which is not the case in our approach. Although we have used the same approach for reordering the examinations in difficult and easy sets separately, examinations in different sets can be reordered based on a different heuristic at each iteration.

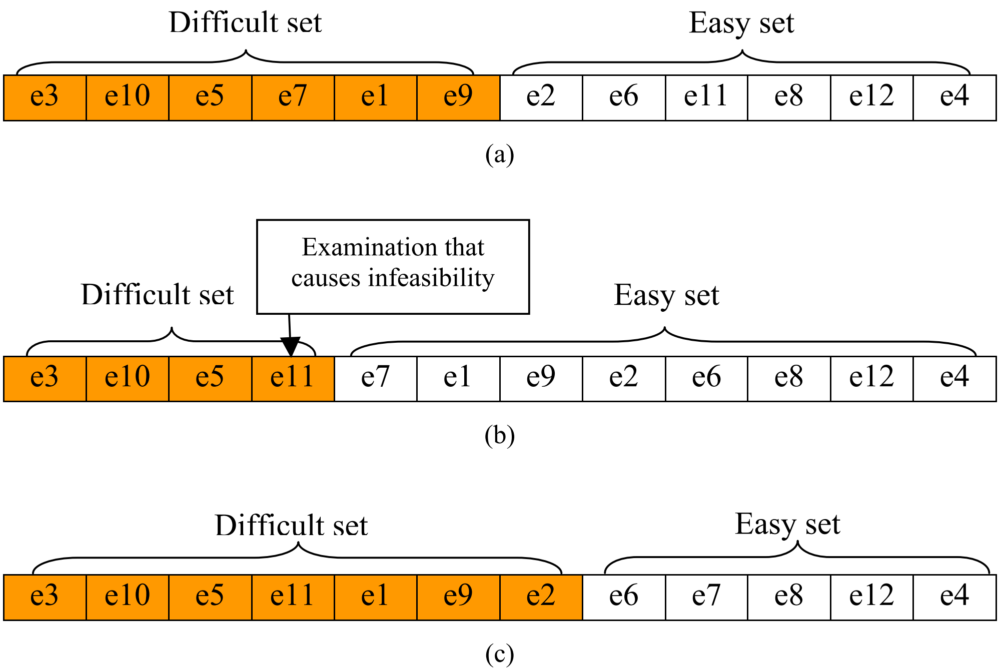


Figure 3. Difficult and easy sets (a) in the first iteration, (b) after an iteration is over (a) resulting with an infeasible solution, (c) after an iteration is over (a) resulting with a feasible solution.

3 Experiments

The experiments were tested on benchmark problems introduced by Carter et al. (1996) and are publicly available at ftp://ftp.mie.utoronto.ca/pub/carter/testprob/.

In this study, we used version I of the 12 problems that were adapted from Qu et al. (2009) to differentiate various versions of the problem. During the experiments, five runs are performed and the stopping conditioned has been set as 10 000 iteration as to be equal with the experiment done by Qu and Burke (2007). Two types of heuristic ordering for initialisation are investigated: largest degree (LD) and saturation degree (SD). The difficult set is created using these two initial orders are then reordered with either largest degree or saturation degree. In this study, the same heuristic ordering is used for the examinations in the easy set. The heuristics used in a given approach will be denoted by a triplet as [*heuristic used for the initial ordering – heuristic used for ordering the examinations in the difficult set – heuristic used for ordering the examinations in the easy set*] from this point onwards. The size of the boundary set is fixed as 5.

Table 2. Comparing solution quality for (a) [LD-LD-LD], (b) [SD-LD-SD], (c) [LD-SD-LD], (d) [SD-SD-SD] by adding boundary set into difficult set and swapping examinations between boundary and difficult sets with δ=5. (LD = largest degree; SD = saturation degree) (Bold font indicates the best for different ordering and strategy and italic is the best of all for each problem instance).

Problem	**Add the boundary set (δ=5) into the difficult set**				**Swap examinations in the boundary (δ=5) and difficult sets**			
	(a)	**(b)**	**(c)**	**(d)**	**(a)**	**(b)**	**(c)**	**(d)**
car91	5.72	5.60	5.77	**5.44**	5.71	5.37	5.75	***5.34***
car92	4.97	***4.76***	4.90	4.85	5.03	**4.87**	4.95	4.91
ear83 I	***41.36***	41.42	42.53	42.51	**41.90**	42.48	43.38	42.78
hec92 I	12.98	12.76	***12.24***	12.45	13.32	13.15	12.72	**12.52**
kfu93	16.68	16.57	**16.35**	16.40	***16.16***	16.61	16.38	16.49
lse91	13.44	12.96	**12.64**	12.85	13.45	***12.52***	12.93	12.95
rye93	11.13	10.79	***10.23***	10.31	11.35	10.66	10.53	**10.27**
sta83 I	163.93	162.12	**159.32**	159.74	161.98	159.34	159.08	***158.99***
tre92	9.77	9.72	**9.54**	9.69	9.81	9.50	9.66	***9.41***
ute92	30.68	30.08	**29.11**	**29.11**	30.21	29.79	29.34	***28.96***
uta92 I	3.92	**3.78**	3.96	3.87	3.96	***3.77***	3.89	3.82
yor83 I	45.85	46.97	**44.16**	44.75	45.84	46.28	**45.30**	45.39

Table 2 summarises the experimental results obtained applying the proposed approach to the benchmark problem instances. By looking at the best ordering for the difficult set, we observe that the adding boundary set strategy performed better with largest degree ordering where nine out of twelve problem instances has performed significantly better than saturation degree ordering while the swapping strategy has performed better with saturation degree ordering with seven out of twelve problem instances are better compare to largest degree ordering. The best combination ordering for adding boundary set strategy is [LD-SD-LD] while the swapping boundary set strategy performed the best with [SD-SD-SD]. From the perspective of the strategies, it is clear that by swapping the boundary set with the difficult set produced better solution quality as compared to just combining the boundary set as a part of the difficult set. The swapping strategy has obtained seven better results while combining strategy produced the better results for only five problem instances.

Table 3. Comparing solution quality for (a) [LD-LD-LD], (b) [SD-LD-SD], (c) [LD-SD-LD], (d) [SD-SD-SD] with shuffling strategies of adding the boundary set into the difficult set and swapping examinations between the boundary and difficult sets with δ=5 and includes roulette wheel selection for examinations with *n*=3. (LD = largest degree; SD = saturation degree) (Bold font indicates the best for different ordering and strategy and italic is the best of all for each problem instance).

Problem	**Add the boundary set (δ=5) into the difficult set + Roulette wheel selection (*n*=3)**				**Swap examinations in the boundary (δ=5) and difficult sets + Roulette wheel selection (*n*=3)**			
	(a)	**(b)**	**(c)**	**(d)**	**(a)**	**(b)**	**(c)**	**(d)**
car91	5.67	***5.28***	5.64	5.43	5.67	5.57	5.77	**5.48**
car92	4.98	4.91	4.81	***4.76***	4.90	4.95	**4.86**	4.89
ear83 I	41.29	***40.60***	41.39	42.74	**41.67**	42.24	42.14	42.51
hec92 I	***12.09***	12.36	12.45	12.70	**12.20**	12.97	12.38	12.48
kfu93	16.25	16.22	***16.04***	16.43	16.43	16.07	16.20	**16.05**
lse91	12.70	***12.03***	12.76	12.67	13.06	12.75	**12.14**	12.82
rye93	10.52	**10.25**	10.40	10.41	10.45	10.45	***10.20***	10.54
sta83 I	160.20	162.26	**158.68**	160.00	158.60	160.43	***158.39***	161.75
tre92	9.31	9.72	***9.08***	9.55	9.43	9.79	**9.21**	9.63
ute92	**27.81**	27.93	28.57	27.90	28.01	27.84	***27.30***	27.55
uta92 I	3.95	***3.73***	3.83	3.81	3.88	3.92	3.89	**3.74**
yor83 I	45.48	45.24	45.76	**44.36**	545.04	***44.33***	44.51	44.81

In the next set of experiments, the effect of incorporating the roulette wheel into the examination selection process is tested with $n = 3$. As we can see from the results in Table 3, the adding boundary set strategy with roulette wheel selection has performed better by providing eight better solutions as compared to the swapping strategy with roulette wheel selection. From the results, the adding boundary set and selection strategy performed the best with combination of [LD-SD-LD] while the best combination ordering for swapping with selection strategy is [SD-LD-SD]. Comparing the best results obtained from the strategies without roulette wheel selection in Table 2 and the strategies with roulette wheel selection in Table 3, it shows that when incorporating the selection strategy improves the performance of the approach.

Table 4. Comparison of different constructive approaches (LD = largest degree; SD = saturation degree; RWS=roulette wheel selection) (The bold entries indicate the best results for constructive approaches only, while the italic ones indicate the best results for the decomposition approach).

Problem	**Burke & Newall (2004)**	**Carter et al. (1996)**	**Asmuni et al. (2009)**	**Abdul Rahman et al. (2009)**	**Qu & Burke (2007)**	**SD-LD-SD(RWS)**
car91	**4.97**	7.10	5.29	5.08	5.45	*5.28*
car92	**4.32**	6.20	4.54	4.38	*4.5*	4.91
ear83 I	36.16	36.40	37.02	38.44	*36.15*	40.60
hec92 I	11.61	**10.80**	11.78	11.61	*11.38*	12.36
kfu93	15.02	**14.00**	15.80	14.67	*14.74*	16.22
lse91	10.96	**10.50**	12.09	11.69	*10.85*	12.03
rye93	-	**7.30**	10.38	9.49	-	10.25
sta83 I	161.90	161.50	160.40	157.72	***157.21***	162.26
tre92	**8.38**	9.60	8.67	8.78	*8.79*	9.72
ute92	27.41	**25.80**	28.07	26.63	*26.68*	27.93
uta92 I	**3.36**	3.50	3.57	3.55	*3.55*	3.73
yor83 I	40.88	41.70	**39.80**	40.45	42.2	45.24

Table 4 compares our best results obtained from the strategy of roulette wheel selection to the other previous results based on constructive approaches. Given by Qu and Burke (2007) is the closest comparison to our approach as they have implemented a decomposition strategy as well. Comparing the solutions across all problem instances, it is observed that our approach does not yield the best results. However, it provides one better result when compared to the approach proposed by Qu and Burke (2007) for car91. Moreover, we have obtained better results than the approach by Asmuni et al. (2009) for four problems (car91, lse91, rye93 and ute92), Carter et al. (1996) for two problems (car91, car92), respectively. However, Burke and Newall (2004) and Qu and Burke (2007) do not provide the result for rye93.

The overall results once again highlight the importance of the methodology used to change the ordering of difficult examinations, particularly the ones causing infeasibility. In our approach, the ordering of the examinations within the difficult set with respect to the others seems to be vital combined with the assignment strategy. As shown in Figure 4, for the experiments adding and swapping boundary set and difficult set without roulette wheel selection, the average number of the examinations in the difficult set varies with different ordering strategies. The approach using the largest degree ordering generates infeasibility more often for a given solution during the time-slot assignments as compared to the one using the saturation degree ordering. On the other hand, saturation degree ordering might easily create a feasible solution for some problem instances (e.g. car91 and uta92 I). However, using the saturation degree alone does not guarantee a good solution quality.

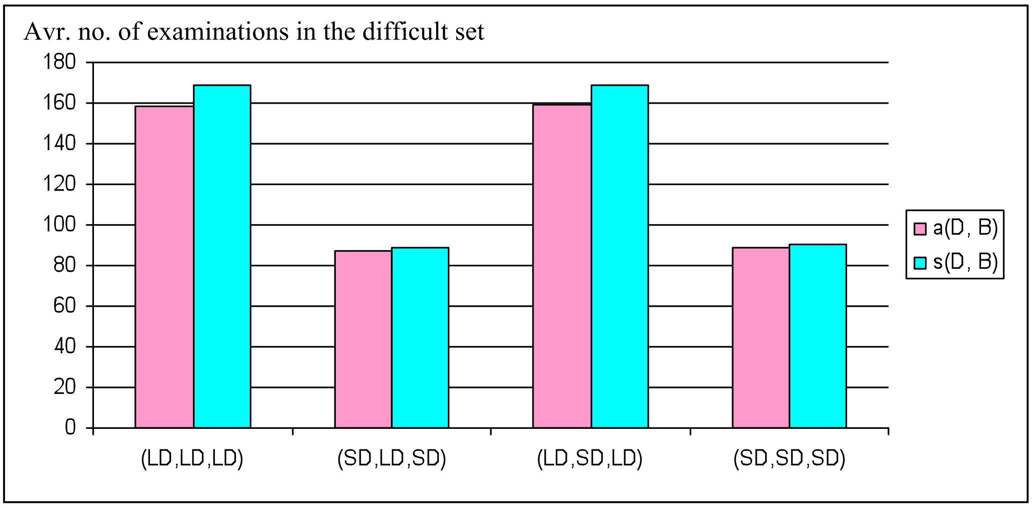
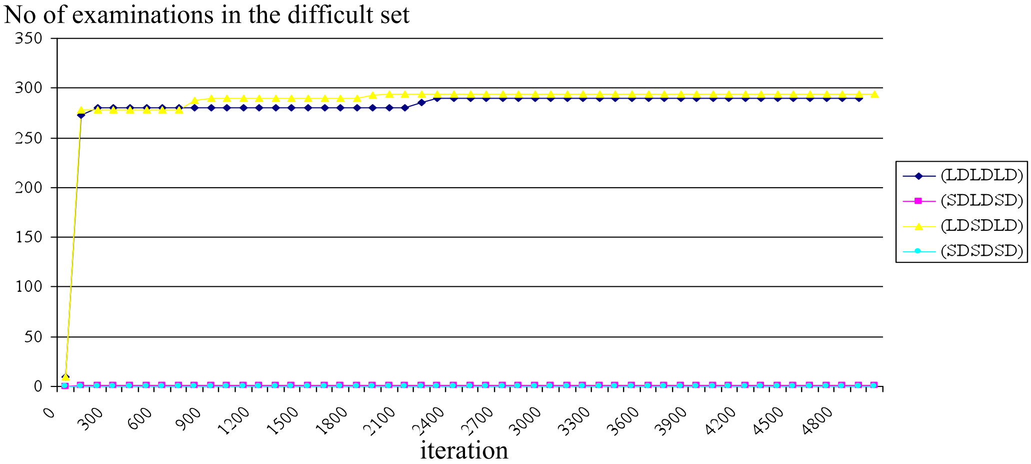


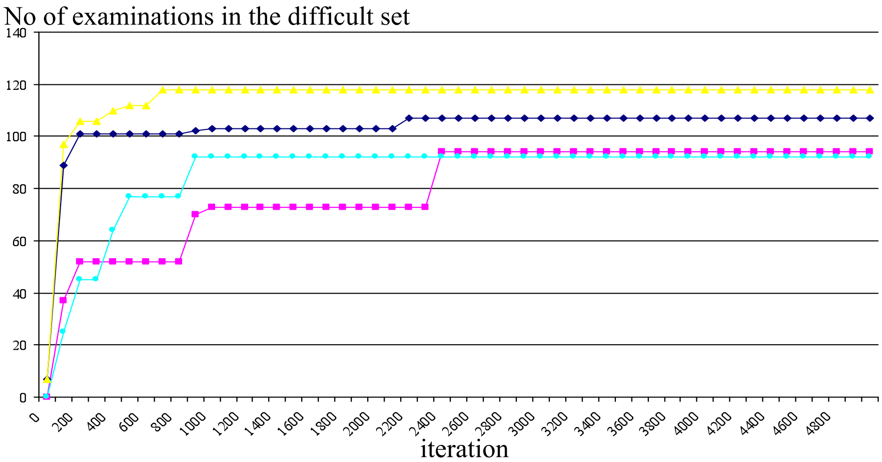
Figure 4. Average number of examinations in the difficult set (its size) over all problems considering all shuffling strategies using different initialisation and reordering heuristics. (LD=largest degree, SD=saturation degree, B=boundary set, D=difficult set, a=add, s=swap).

In some cases, using the saturation degree ordering may easily create a feasible solution when adding or swapping with the boundary set, the infeasible examinations can be obtained in this approach since this approach gives priority of ordering the difficult set. Consequently, adding or swapping the boundary set with the difficult set might have increased the number of examinations in the difficult set.

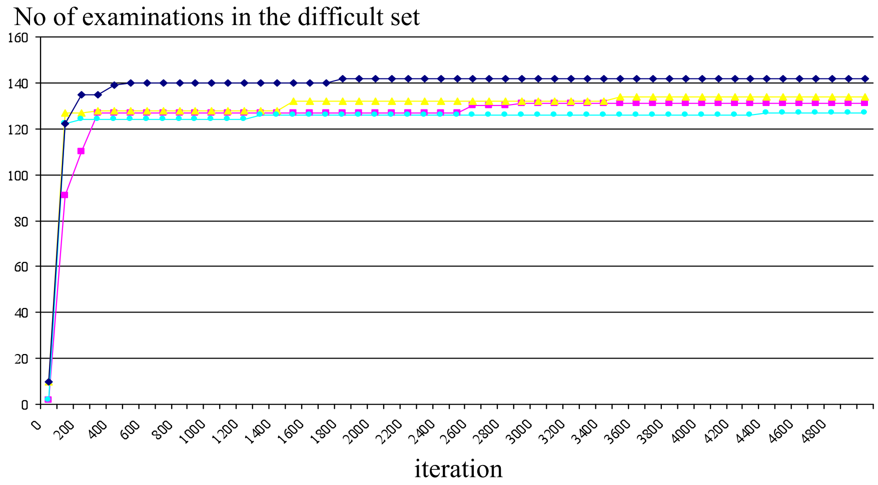
Figure 5((a), (b), (c)) illustrate the number of infeasible examinations and the solution quality at each 100 iteration for different combination of initial ordering and reordering heuristics for the difficult set for car91, kfu93 and yor83 I, respectively. It shows that using largest degree causes increasing number of examinations to generate infeasible solution when compared to the saturation degree. car91 has an obvious difference in the number of infeasible examinations when comparing with the other two types of ordering i.e. [LD-LD-LD] and [LD-SD-LD]. In the other problem, kfu93 and yor83 I the number of infeasible examinations for different ordering is approximately the same but still using [SD-LD-SD] and [SD-SD-SD] are slightly advantageous. In all problems, the number of infeasible examinations is converged to a steady state after some point.



(a) car91



(b) kfu93



(c) yor83 I

Figure 5. The change in the size of the difficult set and the solution quality at every 100 iteration during the sample runs for (a) car91, (b) kfu93, (c) yor83 I. (LD= largest degree, SD= saturation degree, sol= solution quality).

4 Conclusion

This study discusses a novel approach based on adaptive strategies that decomposes the examinations in a given problem into two sets: a set of difficult to schedule and a set of easy to schedule examinations. This decomposition is performed automatically at each iteration, and is augmented with suitable ordering of examinations within each set. In this study, it is observed that by merging or swapping the boundary set with the difficult set could enlarge the search space of a solution and it could improve the solution quality. A stochastic component based on roulette wheel selection is embedded into the approach in order to shuffle the order of examinations. This mechanism gives a higher chance to an examination with a higher score to be selected for timetabling. It is observed that using saturation degree could decrease the possibility of creating infeasible solution and that dynamic ordering gives better ordering of examinations in the list. This preliminary study shows that the proposed approach is simple to implement, yet it is competitive to the other previous constructive approaches. In this study, the same ordering heuristics are used for reordering the examinations in the difficult and easy sets. In fact, the proposed framework allows the use of different strategies. As future work, different strategies will be investigated for reordering of examinations and choosing the examinations from the difficult set.

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