The roots of Granular Computing

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***Abstract*—Granular Computing arose as a synthesis of insights into human-centred information processing by Zadeh in the late '90s and the Granular Computing name was coined, at this early stage, by T.Y Lin. Although the name is now in widespread use, or perhaps because of it, there are calls for a clarification of the distinctiveness of Granular Computing against the background of other human-centred information processing paradigms. This study examines the basic motivation for information granulation and casts Granular Computing as a structured combination of algorithmic and non-algorithmic information processing that mimics human, intelligent synthesis of knowledge from information.**

***Index Terms* — Granular Computing, Human-centred information processing, Information Abstraction, Universal Turing Machine.**

I. Introduction

Granular Computing (GrC), as defined in the outline of the IEEE-GrC'2006 conference information, is a general computation theory for effectively using granules such as classes, clusters, subsets, groups and intervals to build an efficient computational model for complex applications with huge amounts of data, information and knowledge. Though the label is relatively recent, the basic notions and principles of granular computing, though under different names, have appeared in many related fields, such as information hiding in programming, granularity in artificial intelligence, divide and conquer in theoretical computer science, interval computing, cluster analysis, fuzzy and rough set theories, neutrosophic computing, quotient space theory, belief functions, machine learning, databases, and many others. In the past few years, we have witnessed a renewed and fast growing interest in GrC. Granular computing has begun to play important roles in bioinformatics, e-Business, security, machine learning, data mining, high-performance computing and wireless mobile computing in terms of efficiency, effectiveness, robustness and uncertainty.

With the vigorous research interest in the GrC paradigm [3-10, 13-15, 19-22, 25-32] it is natural to see that there are voices calling for clarification of the distinctiveness of GrC from the underpinning constituent disciplines and from the other computational paradigms proposed for large-scale/complex information possessing. Recent contributions by Yao [26-30] attempt to bring together various insights into GrC from a broad spectrum of disciplines and cast the GrC framework as a structured thinking at the philosophical level and structured problem solving at the practical level.

In this study we look at the roots of Granular Computing by elaborating on the original insight of Zadeh [31] stating, "… fuzzy information granulation in an intuitive form underlies the human solution (to the problem). What this suggests is that no machine could solve the problem without using, as humans do, the machinery of fuzzy information granulation. How this could be done is a challenge that has not as yet been met".

We suggest that the above insight has strong foundations in axiomatic set theory, theory of computability and some recent research results linking intelligence to physical computation [1-2]. In fact, re-examining human information processing in this light brings Granular Computing from a question of philosophy to one of physics and set theory.

II. Why Granulation is Necessary?

One of the frequently asked questions about Granular Computing is whether it adds anything new to the established theory and practice of data clustering methods. To answer this question we must first reflect on the meaning of "granulation". The commonly accepted definition of granulation as grouping together of elements based on their indistinguishability, similarity, proximity or functionality serves well the purpose of constructive generation of granules but it does little to differentiate granulation from clustering. To do so, we need to look beyond the algorithmic generation of granules and reflect on the semantics of granular entities.

As a first step, let us draw a parallel between the formation of information granules and formation of subsets from some universal set in a given problem domain. If the granules (subsets) are interpreted uniformly as sets, within the "intuitive set theory" proposed by Cantor, it is inevitable that one arrives at inconsistencies (paradoxes) such as "cardinality of set of all sets" (Cantor) or "definition of a set that is not a member of itself" (Russel). Early attempts at overcoming these inconsistencies were focused on defining axioms that would prevent the occurrence of inconsistencies (Zermelo-Frankel set theory) but this was achieved at the expense of introducing axiom schemas, implying the need for infinitely many axioms to have a consistent set theory. While it is fully acknowledged that the Zermelo-Frankel (ZF) set theory, and its many variants, has advanced our understanding of cardinal and ordinal numbers and has led to the proof of the property of "well-ordering" of sets (with the help of an additional "Axiom

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of Choice") (ZFC), the theory seems unduly complex for the purpose of set-theoretical interpretation of information granules.

A different approach to the axiomatization of set theory designed to yield the same results as ZF theory but with a finite number of axioms (i.e without the reliance on axiom schemas) has been proposed by von Neumann in 1920 and subsequently has been refined by Bernays in 1937 and Goedel in 1940. The defining aspect of Von Neumann-Bernays-Goedel set theory (NBG) is the introduction of the concept of "class" in addition to the concept of "set" [12]. The membership relation

$$a \in b$$

is only defined if a is a set and b is either a set or a class. This is frequently emphasized by a double definition of the membership relation as

$$a \in b \text{ and } a \in C$$

where a, b are sets and C is a class.

The NBG theory prompts a powerful insight into the essence of granulation namely that **the granulation process transforms the semantics of the granulated entities**. Thus the paradoxes that arise in the context of the intuitive set theory are avoided in NBG because the direct comparison of semantically different entities is not meaningful.

The operation on classes in NBG is entirely consistent with the operation on sets in the intuitive set theory. The principle of abstraction implies that classes can be formed out of any statement of the predicate calculus, with the membership relation. Notions of equality, pairing, subclass and such, are thus matters of definitions (a specific abstraction of a formula) and not of axioms. In NBG a set represents a class if every element of the set is an element of the class. Consequently, there are classes that do not have representations, such as the class of all sets that do not contain themselves.

The advantage of NBG is that it provides a framework within which one can discuss a hierarchy of different granulations without running the risk of inconsistency. For instance one can denote a "large category" as a category of granules whose collection and collection of morphisms can be represented by a class. A "small category" can be denoted as a category of granules contained in sets. Thus, we can speak of "category of all small categories" (which is a "large category") without the risk of inconsistency. Similar framework for a set-theoretical representation of granulation is offered by the theory of types published by Russell in 1937, [23]. The theory assumes a linear hierarchy of types: with type 0 consisting of objects of undecided type and, for each natural number n, type $n+1$ objects are sets of type n objects. The conclusions that can be drawn from this framework with respect of the nature of granulation are exactly the same as that drawn from the NBG.

So, answering the original question of this section we conclude that:

- The **concept of granulation is necessary** to denote the semantical transformation of granulated entities;
- Granulation interpreted in the context of axiomatic set theory is **very different from clustering**, which is focused on the mere grouping of similar entities;
- The set-theoretical interpretation of granulation enables **consistent** representation of a hierarchy of information granules.

III. ABSTRACTION AND COMPUTATION

Having established an argument for semantical dimension to granulation, one may ask; how is the meaning (semantics) instilled into real-life information granules? Is the meaning instilled through an algorithmic processing of constituent entities or is it a feature that is independent of algorithmic processing?

To answer these questions we need to look at the nature of computation. In mathematical terms, the general form of computation, formalized as a Universal Turing Machine (UTM), is defined as mapping of sets that have cardinality $\aleph_0$ (infinite, countable) onto sets with cardinality $\aleph_0$. The practical instances of information processing, such as clustering of data, typically involve a finite number of elements both in the input and output sets and represent therefore a more manageable mapping of a finite set with cardinality max_1 onto another finite set with cardinality max_2. The hierarchy of computable clustering can be therefore represented as in Figure 1.

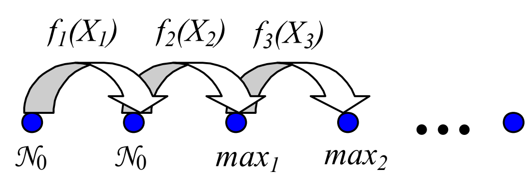

Figure 1. Cardinality of sets in a hierarchy of clusterings implemented on UTM.

The functions $f_1(X_1) \rightarrow X_2$, $f_2(X_2) \rightarrow X_3$, $f_3(X_3) \rightarrow X_4$ represent mappings of:

- infinite (countable) input set onto infinite, countable output set;
- infinite (countable) input set onto finite output set; and
- finite input set onto finite output set, respectively.

The functional mappings, deployed in the process of clustering, reflect the criteria of similarity, proximity or indistinguishability of elements in the input set and, on this basis, grouping them together into a separate entity to be placed in the output set. In other words, the functional mappings generate data abstractions on the basis of pre-defined criteria. We need therefore to understand how these criteria are selected and how they are decided to be appropriate in any specific circumstance. Clearly, there are many ways of defining similarity, proximity or indistinguishability. Some of these definitions are likely to have good real-world interpretation while others may be difficult to interpret or indeed may lead to physically meaningless results.

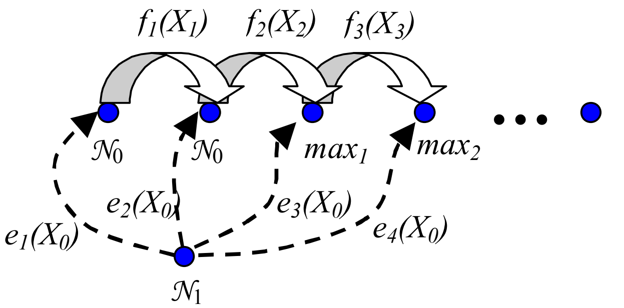

Figure 2. Mapping of abstractions from the real-world domain (cardinality $\mathcal{N}_1$) onto the sets of clusters.

We suggest that the process of instilling the real-world interpretation into data structures generated by functional mappings $f_1(X_1) \rightarrow X_2$, $f_2(X_2) \rightarrow X_3$, $f_3(X_3) \rightarrow X_4$, involves reference to the real-world, as illustrated in Figure 2. This is represented as execution of "experimentation" functions *e(*)*. These functions map the real-world domain X_0, which has cardinality $\mathcal{N}_1$ (infinite, continuum), onto sets X_1, X_2, X_3, X_4, respectively.

At this point, it is important to appreciate that the experimentation functions $e_1(X_0) \rightarrow X_1$, $e_2(X_0) \rightarrow X_2$, $e_3(X_0) \rightarrow X_3$, $e_4(X_0) \rightarrow X_4$, are not computational because their domain have cardinality $\mathcal{N}_1$. So, the process of defining the criteria for data clustering, and implicitly instilling the meaning into information granules, relies on the laws of physics and not on the mathematical model of computation. Furthermore the results of experimentation do not depend on whether the experimenter understands or even knows the laws of physics. Because of that we consider the experimentation functions as providing objective evidence.

IV. Granular Computation

An important conclusion from the discussion above is that the discovery of semantics of information abstraction, referred to sometimes as structured thinking, or a philosophical dimension of granular computing, can be reduced to physical experimentation. This is a very welcome development as it gives a good basis for the formalization of the granular computing paradigm.

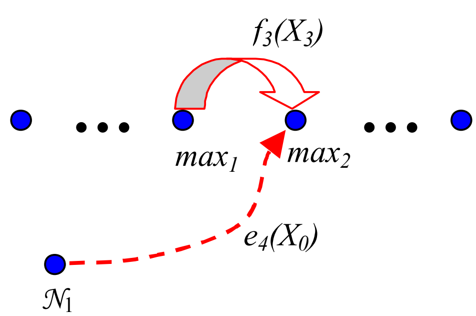

Figure 3. An instance of Granular Computing involving two essential components: algorithmic clustering and empirical evaluation of granules.

We propose that Granular Computing is defined as a **structured combination** of algorithmic abstraction of data and non-algorithmic, empirical verification of the semantics of these abstractions. This definition is general in that it does not specify the mechanism of algorithmic abstraction nor it elaborates on the techniques of experimental verification. Instead, it highlights the essence of combining computational and non-computational information processing. Such a definition has several advantages:

- it emphasizes the complementarity of the two constituent functional mappings;
- it justifies the hyper-computational nature of GrC;
- it places physics alongside set theory as the theoretical foundations of GrC;
- it helps to avoid confusion between GrC and purely algorithmic data processing while taking full advantage of the advances in algorithmic data processing.

A specific instance of Granular Computing is illustrated in Figure 3.

A final question worth addressing here is whether the "structured combination" of two well-established scientific methods (abstraction and experimentation) generates an added value compared to the component methods taken in isolation. Of course, this question is difficult to answer "a priori" - at the formative stage of the discipline. The proof of the value added of GrC will be through the development of insights that could not be derived from the constituent methods taken in isolation. In the meantime, it might be instructive to remember the wisdom that "the whole is frequently greater than the sum of its parts". After all, the knowledge of properties of hydrogen and oxygen does not imply the knowledge of the **combined** molecule - water.

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