A PREDICTIVE MACROSCOPIC CITY TRAFFIC FLOWS SIMULATION MODEL

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1. Abstract.

This paper describes a new predictive macroscopic city traffic flows simulation model that is intended to provide a basis for the real-time optimisation of urban traffic. The model has two components: the first one performs real-time estimation of turning movements coefficients based on the measurements of traffic flows and the second one uses these estimates, together with externally provided traffic light controls, to predict traffic flows. The model is evaluated on historical data containing both traffic flows and traffic controls information.

2. Introduction.

The *early computer assisted traffic control systems* relied on fixed signal timing plans which were developed off-line. The selection of an appropriate timing plan was made either according to the time of the day or based on some traffic measurements. In deriving the sets of timing plans, off-line models such as TRANSYT (Robertson 1969) were commonly used. These systems had only a limited ability to adjust its operation to current traffic conditions thus not taking full advantage of the available road capacity. Any variation of traffic flows, that was not anticipated at the stage of building timing plans, created a disruption that propagated through the network.

With the *current traffic control systems*, the signal timings are evaluated in real time in response to changing traffic conditions. Examples of such real time, demand responsive traffic control systems include the Australian SCATS (Sims 1979), the British SCOOT (Hunt et al 1981) or the Spanish CARS (Grau at al 1992) systems. The characteristic feature of these systems is the prediction of queue length in every link using the measurements of the volume of incoming traffic into each link. The time-scale of each prediction is equal to the travel time between the detector and the back of the corresponding queue and is of the order of the duration of traffic signals cycle. Since the predictions are used in the optimisation of traffic signal timings they also define the time horizon of optimised control. The improvement of demand responsive control systems over the fixed timing plans systems has been found to be very significant, particularly for high traffic volumes. From the control theory viewpoint, these systems can be seen as including a feed-forward signal which provides an "advance notice" to the control system about the expected length of queues and improves the system performance.

The *future generation traffic control systems* will attempt to provide a supervisory layer of optimised control by using both in-vehicle and road-side dynamic route guidance. By its very nature, route guidance has to operate on time scales that are significantly longer than a single traffic signals cycle in order to be useful to road users. Consequently, the optimisation horizon of the supervisory control schemes must extend over several traffic signal cycles thus necessitating correspondingly longer-term predictions of the evolution of traffic flows.

This paper presents a novel mathematical model of the evolution describing the macroscopic traffic flows. The model can be used for deriving longer-term predictions of queues and traffic densities in each link using not only the corresponding measurements in a link but also, indirectly, the measurements in 'up-stream' links through the use of the estimates of drivers' turning movements on the intervening intersections.

3. Description of the model.

The prediction of traffic flows and queue lengths in large scale (several hundred nodes) urban traffic networks requires consideration of averaged traffic flows and platoons of vehicles, rather then individual vehicles.

The simulation model has been formulated in a state-space with the state defined as a vector that has two state variables for each link, the queue length at the red-to-green transition time instance "c", denoted as $q_i(t_c^i)$, and the average traffic density during the traffic signals cycle, denoted as $k_i(t_c^i)$. The

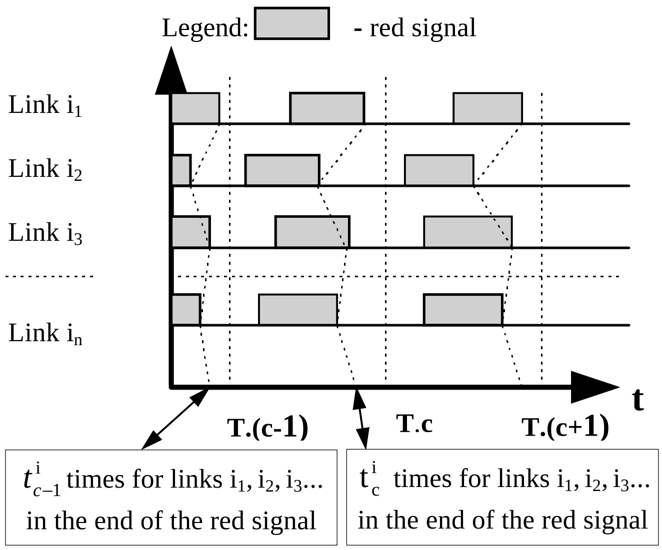

Figure 1. Time Instances Definition

model is unconventional in that the state variables are not evaluated in a single, common instance of time, but in time instances that are link specific and correspond to the occurrence of the longest queues during the traffic signals cycle (Figure 1). Such a choice of the state vector appears to have several advantages. Since the control of traffic signals involves adjusting the cycle time, the offset of cycles on consecutive cross-roads and the split of cycle into red and green times, the model of traffic dynamics that uses the state vector evaluated at the red-to-green transitions, implicitly takes into account all cycle and offset controls and it needs to represent explicitly only the split timings.

a) Queue Length Calculation.

The model calculates recursively the queue length $q_i(t^i_{c+1})$ for link "i" during the red-to-green transition in the cycle "c+1" on the basis of:

- the queue length $q_i(t^i_c)$ in the link "i" in the current cycle "c";
- the number of the cars $Q_{i,c+1}$ joining the queue in link "i" during the cycle "c+1";
- the number of cars $N_{i,c+1}$ discharged from the queue during the green signal in the cycle "c+1", $\Theta_{i,c+1}$.

The evaluation of the actual queue lengths needs to take into account that vehicles arriving during the green signal could cross the stop-line and not necessarily form a queue. The current model assumes a uniform distribution of the arrival rate at the stop line thus implying that if T is the cycle duration and $\Theta_{i,c+1}$ is the green light duration for the link "i" in the simulation step "c+1", the number of cars arriving at the stop-line or joining the back of the queue during the green signal will be

$$Q^{\Theta}_{i,c+1} = Q_{i,c+1}\frac{\Theta_{i,c+1}}{T} \qquad (1)$$

and, the number of the cars arriving at the stop line during the red signal and forming part of the new queue will be

$$Q^{T-\Theta}_{i,c+1} = Q_{i,c+1}\frac{T-\Theta_{i,c+1}}{T} \qquad (2)$$

Three specific cases were considered in order to derive the recursive equation for the state variable $q_i(t^i_{c+1})$. The first case is illustrated in Figure 2*a*. Here the duration of the green light period is not enough to discharge the whole queue $q_i(t^i_c)$. The number of vehicles that are discharged , $N_{i,c+1}$ is defined by the discharge rate d_i of the link and the duration of the green light signal

$$N_{i,c+1} = d_i \cdot \Theta_{i,c+1} \qquad (3)$$

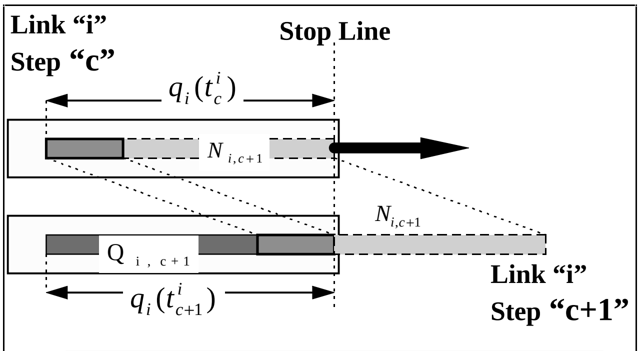

Figure 2a. Queue Length Definition. Case 1.

Consequently the expected queue in the time instance t^i_{c+1} is

$$q_i(t^i_{c+1}) = q_i(t^i_c) + Q_{i,c+1} - N_{i,c+1} \qquad (4)$$

The second case, illustrated in Figure 2b, concerns the situation when the queue $q_i(t^i_c)$ is smaller then the maximum number of cars that can get discharged during the green signal but the number of cars that actually cross the stop-line is, as before, defined by the product of the discharge rate of the link and the duration of the green light. (see equation (3)). This is due to the fact that some of the cars that arrive at the stop-line during the green signal are able to cross without forming a queue. Consequently the new queue $q_i(t^i_{c+1})$ is formed by those cars that arrive during the green signal but, because of the finite discharge rate d_i, were not able to cross the stop-line and the cars that arrive during the red signal

$$q_i(t^i_{c+1}) = [Q^{\Theta}_{i,c+1} - (N_{i,c+1} - q_i(t^i_c))] + Q^{T-\Theta}_{i,c+1} \qquad (5)$$

bearing in mind that $Q_{i,c+1} = Q^{\Theta}_{i,c+1} + Q^{T-\Theta}_{i,c+1}$ the new queue is as in case 1

$$q_i(t^i_{c+1}) = q_i(t^i_c) + Q_{i,c+1} - N_{i,c+1} \qquad (6)$$

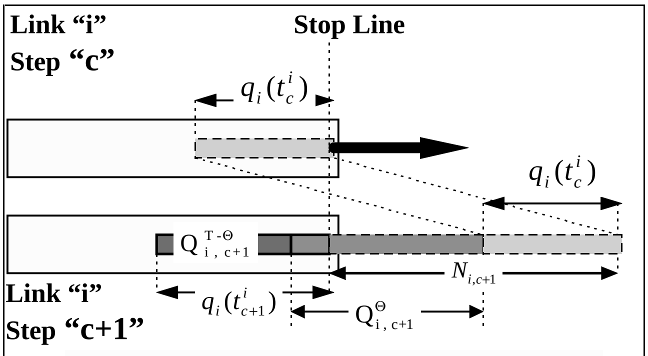

Figure 2b. Queue Length Definition. Case 2.

The third case, illustrated in Figure 2c, is similar to case two in that the queue $q_i(t^i_c)$ is smaller than the maximum number of cars that can be discharged during the green signal. However, all the vehicles that arrive during the green signal also cross the stop-line. This case represents the general situation when the number of cars that actually cross the stop-line is less than the maximum number defined by the equation (3). The following equations describe the number of discharged cars

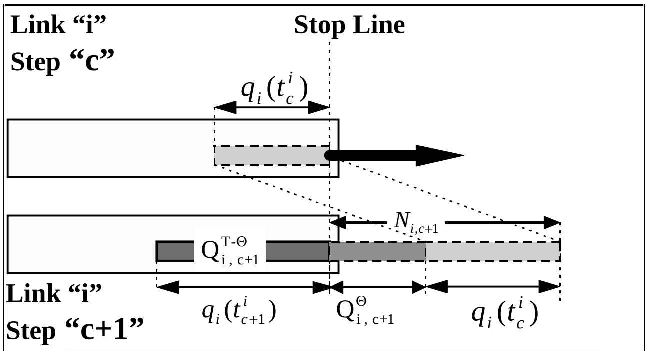

Figure 2c. Queue Length Definition. Case 3.

$$N_{i,c+1} = q_i(t_c^i) + Q^{\Theta}_{i,c+1} \quad (7)$$

and the queue length

$$q_i(t_{c+1}^i) = Q^{T-\Theta}_{i,c+1} \quad (8)$$

Equations (3) - (8) can be combined to give general expressions covering the above three cases

$$q_i(t_{c+1}^i) = Q^{T-\Theta}_{i,c+1} + \max\left\{ q_i(t_c^i) + Q^{\Theta}_{i,c+1} - d_i.\Theta_{i,c+1},\ 0 \right\} \quad (9)$$

and

$$N_{i,c+1} = \min\left\{ q_i(t_c^i) + Q^{\Theta}_{i,c+1},\ d_i.\Theta_{i,c+1} \right\} \quad (10)$$

b) Traffic Density Calculation

The second state variable for link "i", traffic density $k_i(t_{c+1}^i)$ is defined as

$$k = \frac{\text{number of cars in a link at time instance } t_{c+1}^i}{\text{length of a link "i"}} \quad (11)$$

The numerator in (11) represents the combined count of vehicles queuing on the stop-line, $q_i(t_{c+1}^i)$, and the vehicles that are moving along the link. Assuming $k_i(t_{c+1}^i)$ is uniformly distributed along the link "i" the recursive equation for $k_i(t_{c+1}^i)$ can be written as follows

$$k_i(t_{c+1}^i) = k_i(t_c^i) + \frac{B_{i,c+1} - N_{i,c+1}}{l_i} \quad (12)$$

where

- $k_i(t_c^i)$ is the traffic density in the link during the previous cycle "c",
- l_i is the length of the link "i",
- $B_{i,c+1}$ is the number of the cars entering the link during the cycle "c+1" (note that $B_{i,c+1}$ is well approximated by $Q_{i,c+1+a}$ where "a" is an integer representing the travel time along the link expressed in cycles) and
- $N_{i,c+1}$ is the number of cars discharged from the queue for the duration of the green signal $\Theta_{i,c+1}$(calculated using equation (10)).

The way the value of $B_{i,c+1}$, in the equation (12), is calculated depends on whether the link "i" is a "boundary" or an "internal" link (see Figure 3).

For the "boundary" links the model relies on historical data concerning the queue lengths in the previous cycle. This can include data from several most recent cycles but also data from the same time instance on previous days, weeks etc. Current model employs a heuristic function that weights 15 most recent measurements to produce an estimate of $B_{i,c+1}$.

$$B_{i,c+1} = F_q(Q_{i,c}, Q_{i,c-1} \ldots Q_{i,c-14}) \quad (13)$$

The enhanced implementation of the model will include terms from the previous days to instil into the ARMA model the "awareness" of the diurnal variation of the traffic.

For the "internal" links the value of $B_{i,c+1}$ is calculated using the exit flows from the adjacent "upstream" links

$$B_{i,c+1} = \sum_{j\in\Omega_i} N_{j,c}\ .\ m_{ji}(c) \quad (14)$$

where

- Ω_i is a set of indexes representing all "upstream" links adjacent to link "i",
- $N_{j,c}$ is the number of the cars exiting the link "j" and
- $m_{ji}(c)$ is a turning movement coefficient determining what proportion of cars exiting link "j" in cycle "c", $N_{j,c}$, enters link "i" (see Section 4).

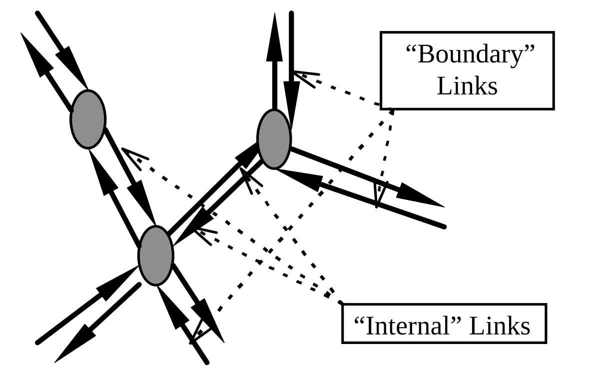

Figure 3. "Boundary" and "Internal" Links

Substituting (14) into (12), the recursive equation for traffic density, $k_i(t_{c+1}^i)$, in an "internal" link is as follows

$$k_i(t_{c+1}^i) = k_i(t_c^i) + \frac{(\sum_{j\in\Omega_i} N_{j,c}\ .\ m_{ji}(c))\ -\ N_{i,c}}{l_i} \quad (15)$$

Similarly, substituting (13) into (12) the recursive equation for traffic density, $k_i(t_{c+1}^i)$, in a "boundary" link is as follows

$$k_i(t_{c+1}^i) = k_i(t_c^i) + \frac{F_q(B_{i,c}, \ldots B_{i,c-14})\ -\ N_{i,c}}{l_i} \quad (16)$$

4. Turning Movements Estimation.

Identification of turning movements coefficients from flow counts has been extensively studied by various researchers (Kessaci 1988, Cremer 1983 and 1987). The proposed methods broadly fall into four classes: Least Squares estimation involving cross-correlation matrices; constrained optimisation methods; methods using simple recursive estimation formulas and Kalman filtering estimation.

For a four-way intersection with measured bi-directional flows in the links (Figure 4), a system of four simultaneous equations, representing a balance of flows into each of the four links, can be formulated. Each equation contains three unknown turning movement coefficients. Given that the coefficients from each link add-up to a unit one can write a system of eight simultaneous equations with twelve unknowns:

$$\sum_{j=1}^{4} N_{j,c}.m_{ji} = B_{i,c} \qquad \text{for i = 1,2,3,4} \quad (17)$$

$$\sum_{j=1}^{4} m_{ij} = 1 \qquad \text{for i = 1,2,3,4} \quad (18)$$

(it is assumed that there are no "U" turns thus $m_{ii} = 0$ for i = 1,2,3,4).

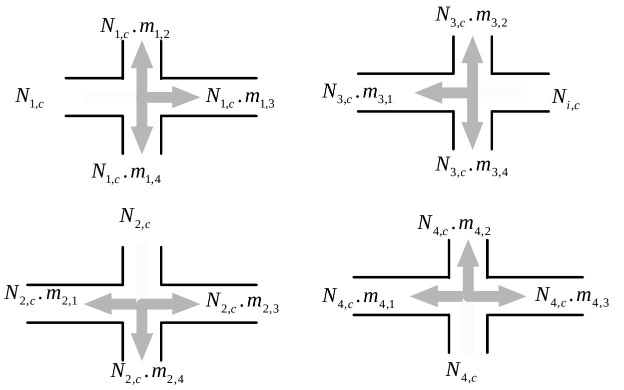

Figure 4. Splitting coefficients $m_{i,j}$ for 4-way intersection

Clearly, this system of equations is under-determined and it needs to be augmented by additional equations before a unique set of turning movement coefficients can be found. The additional equations represent the measurements of inflows into the links during individual signalling stages within a cycle. However, only these links can be considered for which the metering loops are positioned immediately after the intersection, so that the variation in the vehicle dynamics does not distort the measurements significantly. The equations that take into account the "stage" flows, $B_{i,c}^{\text{Stage}}$, can be written as follows

$$\sum_{j \in \Omega_j^{Stage}} N_{j,c} \,.\, m_{ji} = B_{i,c}^{Stage} \qquad i \in \Omega_i^{Stage} \tag{19}$$

where

Ω_j^{Stage} is a set of indexes representing the links from which the traffic is allowed to flow during a given stage, and

Ω_i^{Stage} is a set of indexes representing the links into which the traffic can flow during a given stage,

The resulting system of equations (17)-(19) is over determined and it can be solved using least square estimation to give a set of turning movement coefficients in a single cycle. However, because of the random nature of the traffic flows it is more meaningful to consider averaged turning movement coefficients calculated on the basis of a "moving window" of 8 to 15 signalling cycles. In the current implementation we have directly applied the least squares estimator to the set of equations (17) - (19) written for 9 consecutive cycles. Sample results are presented in the following section.

5. Results.

The predictive macroscopic traffic flows simulation model was applied to the SCOOT controlled traffic network in Mansfield, Nottinghamshire. The measurement data was collected via direct modem connection to SCOOT. The results of simulation experiments are presented in Figures 5. and 6. Figures 5.a-b represent $q_i(t_c^i)$ predictions for one of the "internal" links with different prediction time horizon and are compared to real data collected by SCOOT. The predictions and the real data are averaged on the basis of 9 cycles. The experiments showed that the current model achieves greater accuracy of predictions for "internal" links than for the "boundary" links. In both cases the precision of the simulated data stays below 20% (for up to 4 cycles ahead predictions which are averaged for 9 cycles). Figures 6*a-b*. represent m_{ij} coefficients estimated using the same data set for a 3-way intersection.

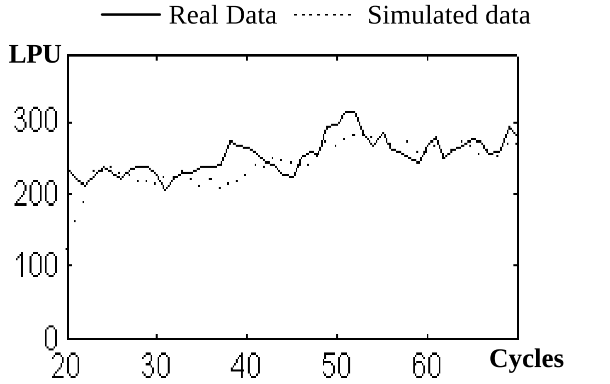

Figure 5-*a*. $q_i(t_c^i)$ prediction.

Prediction Horizon 2 Cycles. Deviation 12%.

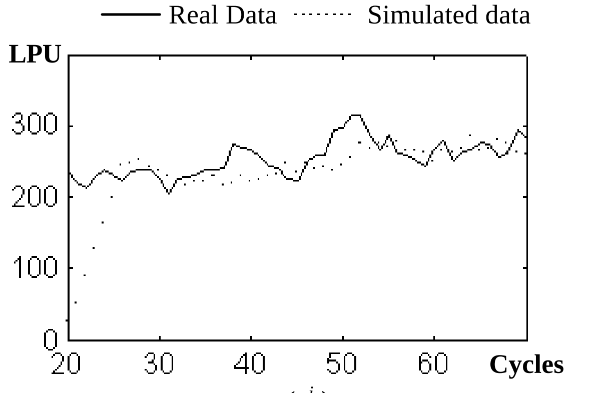

Figure 5-*b*. $q_i(t_c^i)$ prediction.

Prediction Horizon 4 Cycles. Deviation 14%.

6. Conclusions

The macrosimulation model described in this paper has been shown to have potential for application in predictive traffic control schemes. Early evaluation of the model on the real traffic flows data confirms that the state-space formulation, with link dependent discrete time, provides an adequate means for the modelling of macroscopic traffic evolution. It is expected that the accuracy of the macrosimulation model will be enhanced by the inclusion of "previous day" terms in the ARMA model for the estimation of "boundary" inflows.

The current model provides also a baseline against which we intend to assess the impact of the adopted simplifying assumptions, in particular, the impact of the assumption about the uniform distribution of traffic density along the link.

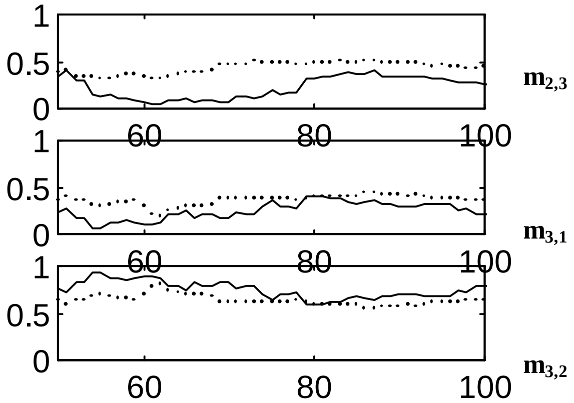

Figure 6. $m_{ji}(c)$ **prediction.**

Prediction Horizon 4 Cycles.
Average deviation 29%.

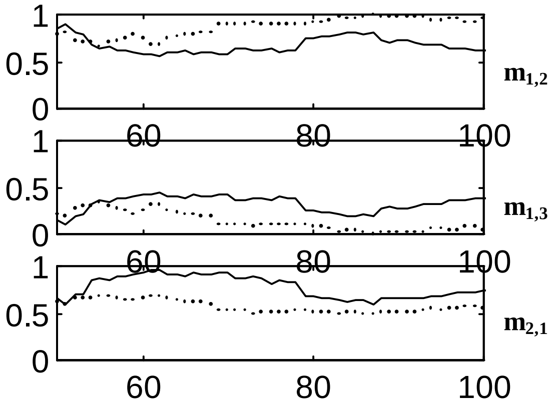

Figure 6. $m_{ji}(c)$ **prediction.**

Prediction Horizon 4 Cycles.
Average deviation 29%.

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