# Leakage reduction by optimised control of valves in water networks

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*The problem of minimisation of leakages due to overpressure in a water distribution network is considered, and an algorithm for computation of the optimal valve controls based on the sparse revised Simplex method is presented. Computational experience indicates that the method has low memory and processor time requirements and is suitable for online implementation on minicomputer systems. The economy of the optimal valve control policy is shown by comparison of the volume of leakages for uncontrolled, manually controlled and optimally controlled networks.*

## 1 Introduction

The primary aim of water distribution control is to maintain sufficient pressure to ensure that all demands, wherever and whenever they occur, can be met. The idealised requirement of system operation is to keep the pressure of the water in each individual node constant, relative to ground level. This is referred to as an optimal head profile. However, owing to the head/flow relationships in the network, the optimal head profile can only be maintained in a few nodes of the network, while in the others the operational pressure remains higher. As the complexity of a distribution network grows, the task of achieving an optimum pressure becomes more and more difficult, and the average overpressure tends to increase. This, in turn, results in an increased energy cost, increased volume of distributed leakages and higher risk of major bursts, particularly during the night period when the pressure rises additionally owing to decrease of consumer demand. In complex networks the volume of leakages can amount to approximately 25% of the total production and consequently represents the main potential for improvement of water distribution system economy.

Minimisation of the overpressures is possible by remote control of valves installed on the pipe network in accordance with the changing demand pattern. However, computation of the optimum valve settings is usually a relatively difficult task owing to the high dimensionality of the optimisation problem and the nonlinearity of the network model. The application of conventional optimisation methods is, consequently, not realistic in view of the computational resources needed and the requirement for real-time control of the water distribution system.

In the present paper, the optimisation problem is expressed in a form which permits application of linear programming optimisation techniques and, in particular, the sparse revised Simplex method is shown to be advantageous. This approach makes it possible to take full advantage of the sparse structure of the problem and to achieve low solution times. A highly sparse factorisation of the basis matrix is maintained using an algorithm proposed by Reid (1975). Studies on networks of different sizes give rise to an estimated computation time, for a network of 100 nodes with 10 control valves, of about 20 s using a Perkin Elmer 3220 minicomputer. An application of the computed control policy is shown to result in a substantial reduction of the distributed leakages in the system.

## 2 Problem formulation

The task of minimisation of the volume of leakages can be seen as a minimisation of discrepancies between a current and an optimal head profile in the network subject to operational limits on the valve controls $v_k$.

$$\min_{v_k} \sum_i | h_i - h_i^0 | \qquad \ldots (1)$$

$$\text{s.t. } 0 < v_k < v_k^{\max}$$

where $i = 1, \ldots, N$ – is the number of network nodes
$k = 1, \ldots, K$ – is the number of control valves.

It is apparent however that, because of the limited number of control valves ($K \ll N$), not all heads $h_i$ can be controlled independently, therefore it is practical to consider only the subset of network nodes which impose the most severe requirements for the water supply system. These are usually the nodes which have locally the highest ground elevation or the biggest load. In effect, the optimisation problem [Eqn (1)] can be expressed as

$$\min_{v_k} \sum_j | h_j - h_j^0 | \qquad \ldots (2)$$

$$\text{s.t. } 0 < v_k < v_k^{\max}$$

where $j = 1, \ldots, R$ is the number of reference nodes in the network.

To be able to perform the optimisation [Eqn (2)] it is necessary to find a functional relationship between nodal heads $h_j$ and valve controls $v_k$. These can be expressed in many different forms depending on the chosen set of state variables. In the case of a water distribution system it is convenient to select the heads in all network nodes and inflows in fixed head nodes as state variables in order to enhance the preservation of sparsity in the mass balance equations

$$\sum_{j \in M_i} f_{ij}(\mathbf{h}) = b_i \qquad i = 1, \ldots, L \qquad \ldots (3)$$

$$\sum_{j \in M_i} f_{ij}(\mathbf{h}) + u_i = b_i \qquad i = L+1, \ldots, N \qquad \ldots (4)$$

$$u_i = u_i^0 \qquad i = L+1, \ldots, N \qquad \ldots (5)$$

where $\mathbf{h} = [h_1, \ldots, h_N]^T$ is a vector of nodal pressures
$u_i$ is the inflow in the fixed-head node
$f_{ij}$ is a head/flow function of $i$–$j$ network element
$M_i$ is a set of nodes incident to node $i$
$b_i$ is a nodal balance.

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For a network containing control valves, the state vector must be extended by the addition of variables representing valve openings. The head/flow function $f_{ij}$ in Eqns (3) and (4) is then replaced by $f_{ij}(\mathbf{h}, v)$ where $v = [v_1, \ldots, v_k]^T$ and $v_k$ is the $k$th valve control. However, $v_k$ is the control variable and is not known in advance. The information about the value of $v_k$ can only be expressed approximately in the following way.

$$v_k + \omega_k^v = v_k^0 \qquad k = 1, \ldots, K \qquad \ldots (6)$$

$$v_k + \nu_k = v_k^{\max} \qquad k = 1, \ldots, K \qquad \ldots (7)$$

where Eqn (6) represents uncertainty about a current approximation of valve control $v_k^0$, and Eqn (8) represents an operational limit of valve control ($\nu_k > 0$). The problem described by Eqns (2)–(7) could be solved by a predictor–corrector type of procedure; however, this would involve a full load flow solution followed by a sensitivity analysis at every stage, implying unnecessarily high computational effort. In the present paper a formulation is proposed which allows for computation of optimal valve controls in a single stage. For this purpose Eqns (3), (4), (5) and (7) are complemented by variables

$\omega_i^L : i = 1, \ldots, L,$

$\omega_i^L : i = L + 1, \ldots, N,$

$\omega_i^u : i = 1, \ldots, N-L,$

$\omega_i^v : i = 1, \ldots, K$

respectively, in a similar form to Eqn (6) except that the values of these additional variables are kept zero. Additionally, equations for the head in the network reference nodes are written as

$$h_j + \omega_j^h = h_j^0 \qquad j = 1, \ldots, R \qquad \ldots (8)$$

where $\omega_j^h$ is a discrepancy between current and optimal head profile. Using the notation introduced above the optimisation problem [Eqn (2)] can now be written

$$\min_{\mathbf{x}} \mathbf{w}^T |\omega| \qquad \ldots (9)$$

s.t. $\mathbf{g}(\mathbf{x}) + \omega = \mathbf{z}$

where $\mathbf{g}(.)$ is a non-linear functional of $\mathbf{x}$.

$$\mathbf{x} = [h_1, \ldots, h_N, u_1, \ldots, u_{N-L}, v_1, \ldots, v_K, \nu_1, \ldots, \nu_K]^T$$

$$\mathbf{w} = [w_1^L, \ldots, w_N^L, w_1^u, \ldots, w_{N-L}^u, w_1^v, \ldots, w_K^v, w_1^{\nu}, \ldots, w_K^{\nu}, w_1^h, \ldots, w_K^h]^T$$

$$\omega = [\omega_1^L, \ldots, \omega_N^L, \omega_1^u, \ldots, \omega_{N-L}^u, \omega_1^v, \ldots, \omega_K^v, \omega_1^{\nu}, \ldots, \omega_K^{\nu}, \omega_1^h, \ldots, \omega_K^h]^T$$

$$\mathbf{z} = [d_1, \ldots, d_N, u_1, \ldots, u_{N-L}, v_1^0, \ldots, v_K^0, v_1^{\max}, \ldots, v_K^{\max}, h_1^0, \ldots, h_K^0]^T$$

The values of the elements of the weighting vector $\mathbf{w}$ are chosen in such a way as to reflect the requirements of the vector $\omega$. Since the mass balance equations and the equations representing operational limits of the valves express physical relationships, the corresponding weights are high, which has an effect of zeroing $\omega_i^L$, $\omega_i^u$ and $\omega_i^{\nu}$. Conversely, the weights corresponding to the equations for valve opening $w_i^v$ are set to zero since the cost of the valve control is neglected in Eqn (2). The equations representing a discrepancy between the current and the optimal head profile are biased with some small positive weights and effectively are the only ones which contribute to the non-zero value of the performance index.

## 3 Solution technique

To cope with the non-linearity of the expressions in Eqn (9) a method of iterative linearisation based on the Newton–Raphson process has been used. This can be summarised as follows

*1.* Expand $\mathbf{g}(\mathbf{x})$ to first order using a Taylor series about an initial guess of the state vector $\mathbf{x}^0$.

$$\mathbf{g}(\mathbf{x}) - \mathbf{g}(\mathbf{x}^0) = \mathbf{J}\,\Delta\mathbf{x} + \omega \qquad \ldots (10)$$

where $\mathbf{J}$ is $P \times Q$. Jacobian matrix
$P$ is a number of equations $P = 2N - L + 2K + R$
$Q$ is a number of variables $Q = 2N - L + 2K$

*2.* Solve the optimisation problem for the linearised constraints.

$$\min_{\Delta\mathbf{x}} \mathbf{w}^T |\omega| \qquad \ldots (11)$$

s.t. $\Delta\mathbf{z} = \mathbf{J}\Delta\mathbf{x} + \omega$

where $\Delta\mathbf{z} = \mathbf{g}(\mathbf{x}) - \mathbf{g}(\mathbf{x}^0)$

*3.* Update the state vector.

$$\mathbf{x}^0 = \mathbf{x}^0 + \Delta\mathbf{x} \qquad \ldots (12)$$

*4.* If $\Delta\mathbf{x}$ satisfies a convergence test then stop, otherwise repeat iteration from *1*.

The estimate of the state vector $\mathbf{x}$ computed according to the Newton–Raphson process generally converges even if the initial guess $\mathbf{x}^0$ is not good. In practice, the initial guess would be the result of the most recent state estimation, and convergence would be achieved in a few steps. The structure of the optimisation problem at stage *2* of the Newton–Raphson process facilitates an efficient solution using the sparse revised Simplex method. To satisfy requirements for non-negativity of the variables and to allow a decrease of the state vector in *3*, the following substitutions are introduced

$$\omega = \mathbf{r} + \mathbf{s} \qquad \ldots (13)$$

where $\mathbf{r} = [r_1, \ldots, r_P]^T$, $\mathbf{s} = [s_1, \ldots, s_P]^T$
$r_i > 0, s_i > 0, r_i + s_i = \omega_i, r_i s_i = 0, i = 1, \ldots, P$

and

$$\Delta\mathbf{x}' = \Delta\mathbf{x} + \mathbf{d} \qquad \ldots (14)$$

$$\Delta\mathbf{z}' = \Delta\mathbf{z} + \mathbf{J}\mathbf{d} \qquad \ldots (15)$$

where

$\mathbf{d} = [d_1, \ldots, d_Q]^T$, and $d_i$ is the maximum decrease of the state variable $x_i$ in one iteration

$$\Delta\mathbf{x}' = [\Delta x_1 + d_1, \ldots, \Delta x_Q + d_Q]^T$$

Now the linear programme can be written in a standard form.

$$\min_{\Delta \mathbf{x}'} \mathbf{w}^T(\mathbf{r} + \mathbf{s}) \qquad \ldots (16)$$

$$\text{s.t.} \quad \Delta \mathbf{z}' = [J : I : -I] \begin{bmatrix} \Delta \mathbf{x}' \\ \mathbf{r} \\ \mathbf{s} \end{bmatrix}$$

where $I$ is a unit matrix.

As the dimension of the basis in the primal Simplex method is determined by the number of equations ($P$), the introduction of variables **r** and **s** does not result in any increase of dimensionality of the problem. Also, the computer-memory requirements remain unchanged as the unit matrices are incorporated implicitly in the Simplex algorithm.

In order to take full advantage of sparsity in the linear programme, the 'elimination' form of basis factorisation has been used. Reid (1975) has proposed an algorithm for the elimination form which also applies a series of row and column permutations to give enhanced sparsity retention. An implementation of this basis handling mechanism is widely available as a routine LA05A in the Harwell subroutine library.

## 4 Computational experience

The performance of the optimal valve control algorithm has been tested on several different sized networks. The detailed results of a study of the effect of incorporation of the control valves and their operation are presented for the 25-node network shown in Fig 1. Parameters of the pipes are given in Table 1. The network contains three pumping

TABLE 1: 25 – node network: pipe parameters

| Line | Length [m] | Diameter [m] | Hazen–Williams coefficient |
|---|---|---|---|
| 23–1 | 606 | 0.457 | 110 |
| 23–24 | 454 | 0.457 | 110 |
| 24–14 | 2782 | 0.229 | 105 |
| 25–14 | 304 | 0.381 | 135 |
| 10–24 | 3383 | 0.305 | 100 |
| 13–24 | 1767 | 0.475 | 110 |
| 14–13 | 1014 | 0.381 | 135 |
| 16–25 | 1097 | 0.381 | 6 |
| 2–1 | 1930 | 0.457 | 110 |
| 3–2 | 5150 | 0.305 | 10 |
| 12–13 | 762 | 0.457 | 110 |
| 15–16 | 914 | 0.229 | 125 |
| 17–16 | 822 | 0.305 | 140 |
| 18–17 | 411 | 0.152 | 100 |
| 20–18 | 701 | 0.229 | 110 |
| 19–17 | 1072 | 0.229 | 135 |
| 20–19 | 864 | 0.152 | 90 |
| 21–20 | 711 | 0.152 | 90 |
| 21–15 | 832 | 0.152 | 90 |
| 22–15 | 2334 | 0.152 | 100 |
| 12–15 | 1996 | 0.229 | 95 |
| 11–12 | 777 | 0.229 | 90 |
| 10–11 | 542 | 0.229 | 90 |
| 8–12 | 1600 | 0.457 | 110 |
| 8–10 | 249 | 0.305 | 105 |
| 9–8 | 443 | 0.229 | 90 |
| 6–8 | 743 | 0.381 | 110 |
| 22–8 | 931 | 0.229 | 125 |
| 22–21 | 2689 | 0.152 | 100 |
| 4–3 | 326 | 0.152 | 100 |
| 5–4 | 844 | 0.229 | 110 |
| 6–3 | 1274 | 0.152 | 100 |
| 5–6 | 1115 | 0.229 | 90 |
| 7–6 | 615 | 0.381 | 110 |
| 5–22 | 1406 | 0.152 | 100 |
| 5–7 | 500 | 0.381 | 110 |
| 6–9 | 300 | 0.229 | 90 |

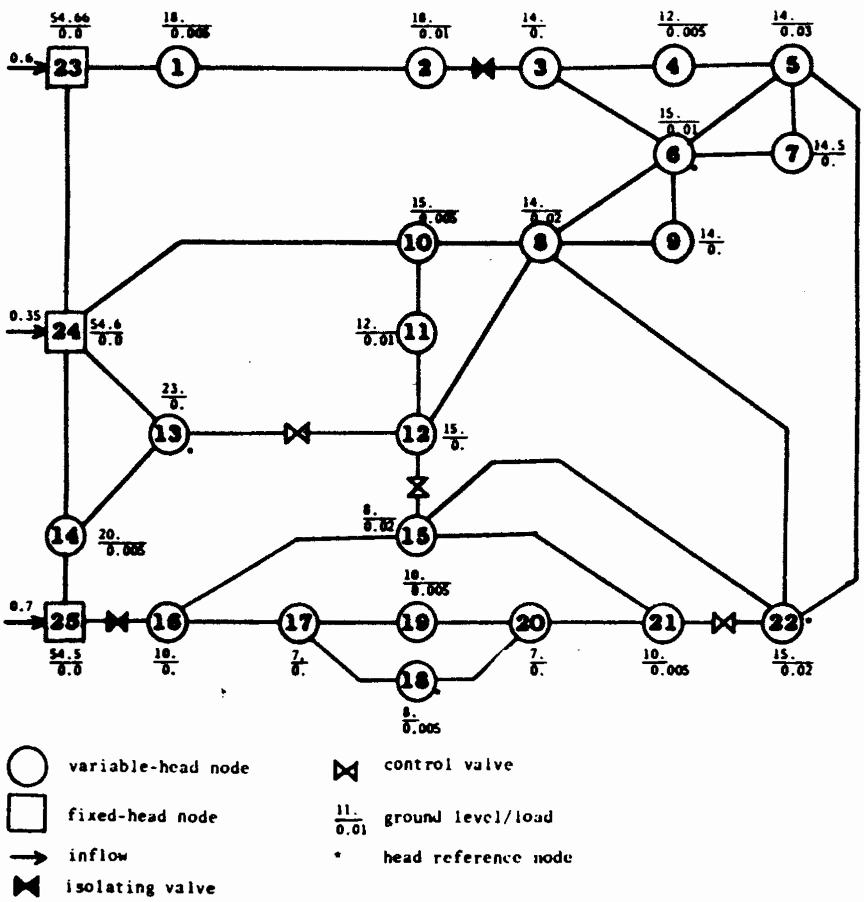


Fig 1 25-node system

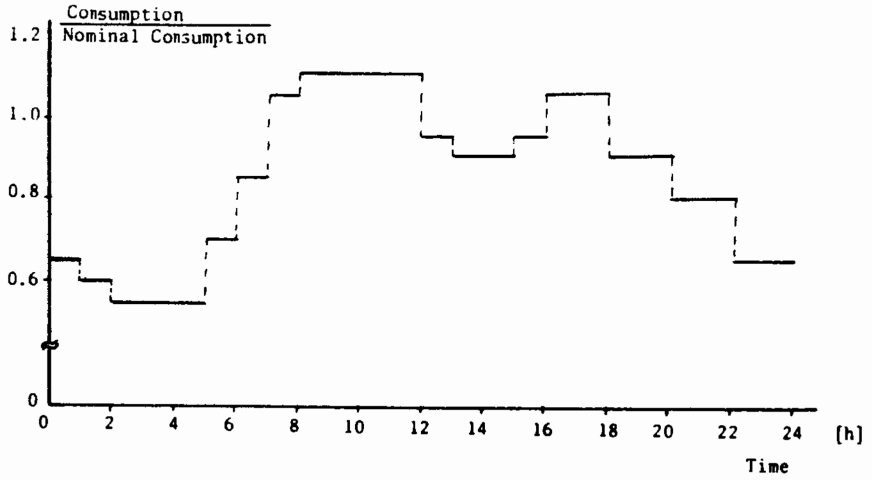


Fig 2 Normalised daily load pattern

stations which are controlled on an on/off basis but can accommodate for ±15% variation of a flow without changing the water supply pressure. Since the variation of the load during the 24-h period is over 200% (Fig 2) it is necessary to combine discrete and continuous control of the pumps. The pumping schedule, presented in Fig 3, has been devised so as to satisfy consumers' demands fully, subject to constraints on the magnitude of the change of flow and the frequency of the on/off control of each individual pump. It is assumed that the hourly load in each of the 15 consumer supply nodes follows the pattern given in Fig 2. The nominal consumption corresponding to '1' in Fig 2 is given alongside the ground level of the node on the network diagram. In order to prevent an excessive service pressure in the nodes which haye a low ground level, two isolating valves (constant throttling) and three control valves are used in pipes 2–3, 25–16 and 12–13, 13–15, 21–22, respectively. The current service pressure is measured in nodes 6, 13, 18 and 22 which have locally the highest ground level.

The volume of leakages has been evaluated for each head profile based on the empirical relationship

$$V = C \sum_i I_i \, h_{Ai}^{1.18} \qquad i = 1, \ldots, S \qquad \ldots (17)$$

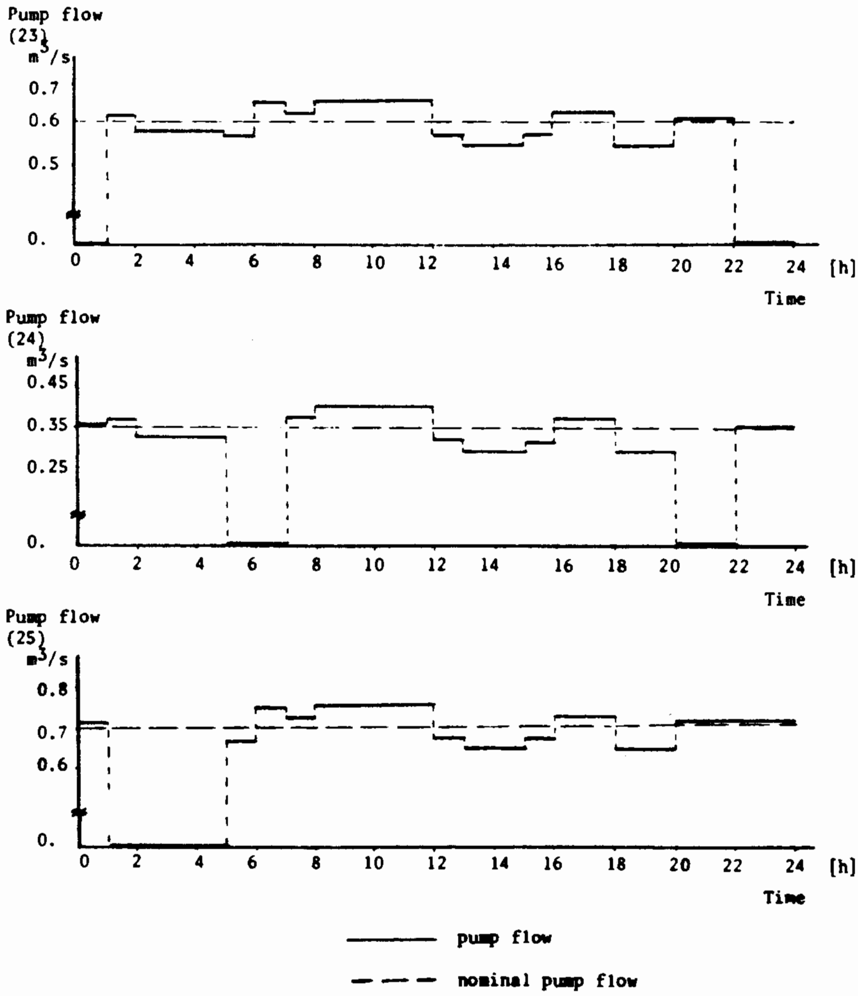


Fig 3 Pumping schedule

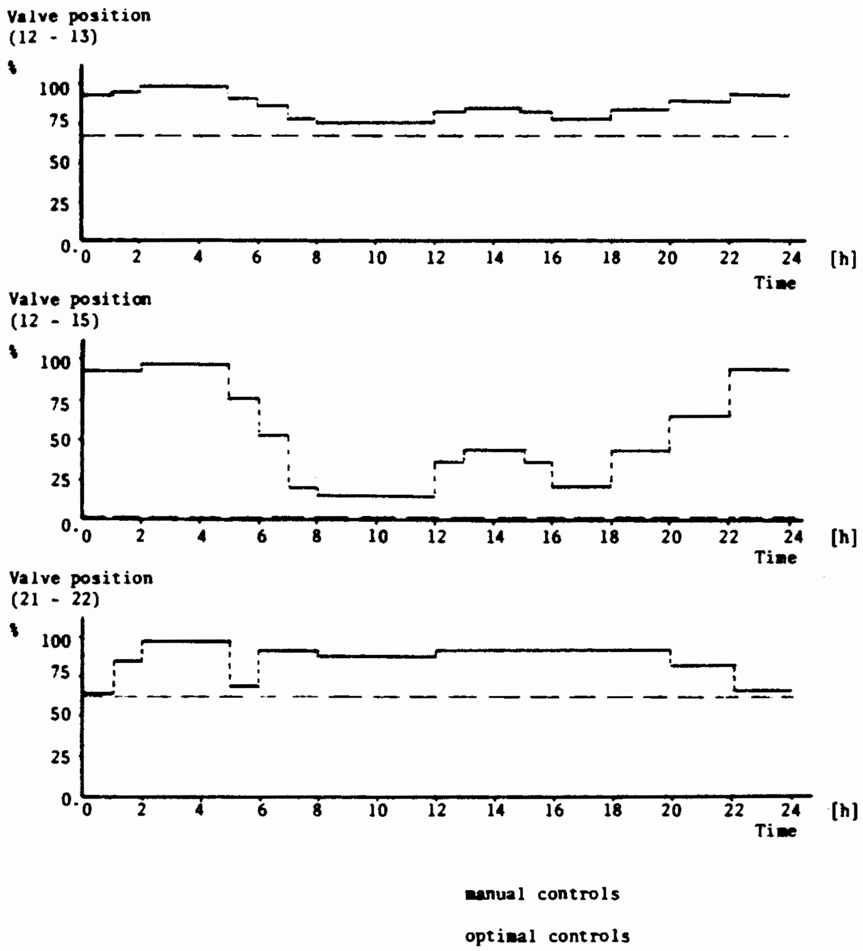


Fig 4 Valve-control strategy for minimal leakage

where $S$ is the number of pipes
$C$ is a constant depending on the network
$l_i$ is the length of the $i$th pipe
$h_{Ai}$ is the average service pressure along the $i$th pipe.

Consequently, for the optimal head profile (30 mAq) the corresponding volume of leakages is $V_{30} = C.30^{1.18} \sum_i l_i$ and the water loss index can be introduced as

$$W = \frac{V - V_{30}}{V_{30}} \qquad \ldots (18)$$

Three schemes of operation of the network have been analysed. In the first case, the network with fully open control valves is considered. The head profile achieved during the operation of the network is identical to that for the network with no control valves. The limit of 30 mAq for the service pressure is exceeded for all loads since the network has to maintain the capability of supplying some emergency loads. Decrease of the consumer load during the night period additionally increases overpressure in the network, which is reflected by the high values of the water loss index given with a dotted line in Fig 5. In the second

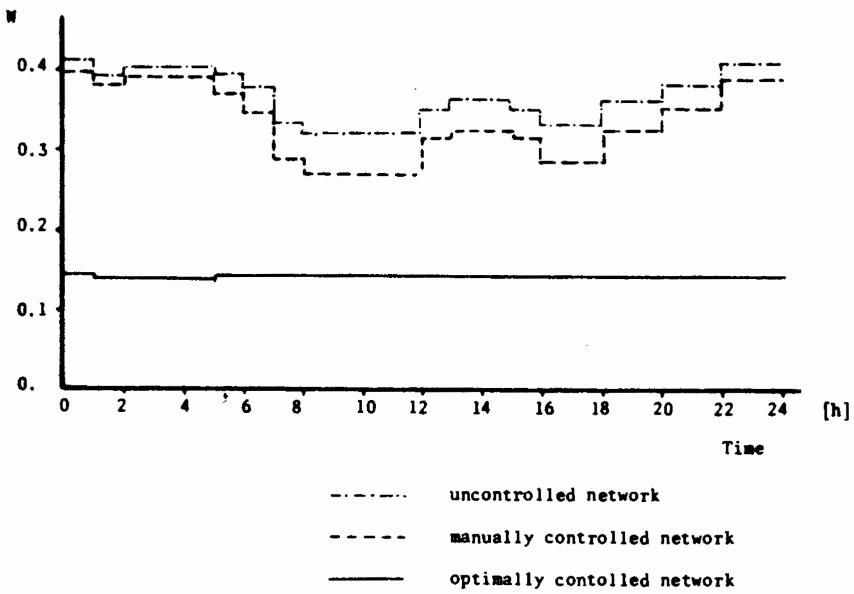


Fig 5 Hourly variation of the water-loss index

TABLE 2: Variation of computation requirements with network size

| | | | | | | |
|---|---|---|---|---|---|---|
| No. of var.-head nodes | 13 | 13 | 13 | 22 | 32 | 100 |
| No. of fixed-head nodes | 3 | 3 | 3 | 3 | 3 | 5 |
| No. of control valves | 1 | 3 | 6 | 3 | 3 | 10 |
| No. of check points | 4 | 4 | 4 | 4 | 4 | 10 |
| No. of state variables | 21 | 25 | 31 | 34 | 44 | 130 |
| No. of equations | 25 | 29 | 35 | 38 | 48 | 140 |
| No. of *N-R* iterations | 5–7 | 5–7 | 5–7 | 5–7 | 5–7 | 5-7* |
| Computation time per *N-R* iter. [*s*] | 0.530 | 0.690 | 0.715 | 0.760 | 1.078 | 3.57* |

* Estimated value

case the control valves are throttled in order to achieve the optimal head profile in the selected reference nodes during the highest daily consumption. This corresponds to the situation in which the network has manually controlled valves which, except in emergency, have constant openings. The area between the dashed and dotted lines in Fig 5 indicates 3.5% reduction of the total leak volume as a result of implementation of such a control policy.

In the third case, the optimal valve controls, shown in Fig 4, have been applied. The discrepancy between the current and the optimal head profile is minimised for the whole range of the consumer loads giving an almost constant value of the water loss index. Application of the optimal control policy results in 19.2% reduction of distributed leakages as compared with the network using manually controlled valves. The non-zero value of the water loss index indicates that by increasing the number of control valves a further reduction of leakages is possible. However, the incremental saving achieved by adding one control valve to the network may be marginal.

The algorithm for computation of the optimal valve controls has been coded in FORTRAN 77 and implemented on a Perkin Elmer 3220 minicomputer with 32-bit word length and floating-point arithmetic. Comparisons of the execution time have been made for 16, 25 and 35-node networks having 1 to 6 control valves. The results are presented in Table 2 and include an estimated computation time for a 100-node network with 10 control valves.

## 5 Conclusions

An algorithm for computation of the optimal valve controls in order to reduce distributed leakages has been presented. Simulation results have indicated a potential for 20% reduction of the volume of leakages using optimised valve control. The algorithm has been shown to be computationally efficient and applicable to online operation using relatively inexpensive computer hardware. The proposed method may also be used at the network planning stage to evaluate the economics of the installation of additional control valves in the network.

## 6 References

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