Minimum norm state estimation for computer control of water distribution systems

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Abstract: The paper presents an algorithm for water-system state estimation in presence of gross errors in measurement data. Application of a modulus norm in place of the more usual least-squares criterion for the minimisation of measurement inconsistency is shown to result in a linear programming formulation. A high computational efficiency for the algorithm is obtained by utilisation of both sparsity techniques and some special features of water distribution systems. Test results are included which demonstrate the applicability of the algorithm for online computer-based state estimation.

List of symbols

A = matrix in LP formulation
b = RHS vector in LP
b^* = RHS vector in optimal solution of LP
B^1 = initial feasible basis
B^k = optimal basis in kth Newton iteration
C = value of performance in state estimation
d = state increment shift
F = reduced Jacobian matrix
F^A, F^R = factors of matrix F
g = functional of nonlinear measurements
I = identity matrix
J = Jacobian matrix
J_i = ith row of J
m = number of measurements
n = number of state variables
N = matrix of nonbasic vectors
s, r = slack variables vectors
w = measurement weight vector
W = modified measurement weight vector
x = state vector
x^0 = initial guess of state vector
Δx = increment of state vector
y = independent variables in LP
z = measurement vector
z^0 = measurement vector for initial guess x^0
Δz = increment of measurement value
α = parameter in Newton method
ω = vector of measurement inconsistency

Subscripts:
i = index of vector element or row in matrix
B, N = indication of basic and nonbasic variables

Superscripts:
A, R = indication of accepted or rejected measurements
T = transpose of matrix
-1 = inverse of matrix
$0, 1, \ldots, k$ = index of iteration in Newton process

Operator:
$|x| = (|x_1|, |x_2|, \ldots, |x_n|)^T$ for $x = (x_1, x_2, \ldots, x_n)^T$

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1 Introduction

The increasing complexity of modern water distribution systems is demanding more widespread application of computerised control schemes. However, any form of advanced control necessitates reliable information about the present state of a system. In reality, the measurements available are limited to a few telemetered values of heads, consumer demands, supplies to the system or pipe flows, which, in themselves, often do not provide a complete picture of system state. Cost considerations frequently severely limit the number of measurements, with the result that many network flows and heads remain unknown. This otherwise inadequate information can, however, be combined with a knowledge of the topology of the distribution network and of the current status of each value to provide a complete picture of flows and heads in the network. Techniques which implement such a combination are known as state estimators, and have been widely utilised in the electrical power monitoring and control field. Provided sufficient basic measurements are available, the state-estimation procedure can also act as a filter for incoming data, which may have been contaminated by meter or communication errors, by producing a comprehensive consistency check. In order to achieve a computationally efficient state estimator, a minimal set of state variables should be selected. In the water distribution systems field it is convenient to select nodal heads and inflows (outflows) at fixed-head nodes as the state variables. Given this information, all other operational parameters, such as pipe flows, can be calculated directly. Furthermore, where the dynamic solution of a network is of interest, an integration procedure can be easily implemented, for example, to provide reservoir volumes. The process of state estimation necessitates a mathematical model of a water distribution network based on conventional head-flow equations, which expresses measurements and pseudomeasurements in terms of state variables. The introduction of pseudomeasurements plays two roles: it enhances the observability of the system, since the real measurements are usually concentrated in some parts of the network leaving other areas devoid of measurements, and, secondly, it provides measurement redundancy, defined as a ratio of the number of measurements and pseudomeasurements to the number of state variables, which facilitates error detection and correction within the estimation procedure.

The most common method of estimation of the state vector from an overdetermined set of measurements is the least-squares method, which gives a minimum-variance estimate provided that the measurements are affected solely by Gaussian noise. Unfortunately, this is rarely the case in online computer control systems, where the measurement inaccuracies are far from a Gaussian distribution but, in fact, contain gross errors derived from telemetry or instrumentation malfunction. A more effective approach is to design a state estimator which will reject erroneous measurements rather than distribute their effect. This can be achieved either by some modifications and extensions of the least-squares method [1, 2] or by replacement of the Euclidean norm by a modulus objective during the minimisation of measurement inconsistency [3]. The present paper develops the latter idea and results in the design of a highly efficient state estimator which is suitable for real-time monitoring and control applications.

2 State-estimation method

Given the set of measurements in generalised form:

$$z = g(x) + \omega \tag{1}$$

where ω represents the measurement inconsistency which is formed by two factors: Gaussian measurement noise and non-Gaussian gross errors (dim $\omega = m$). The state estimation is usually expressed as

$$\min_x C = (z - g(x))^T R^{-1}(z - g(x)) \tag{2}$$

where R is a diagonal measurement noise covariance matrix.

The present paper proposes minimisation of the modulus of measurement inconsistency for computation of the state estimate.

$$\min_x C = w^T | z - g(x) | \tag{3}$$

where w is a measurement weight vector (dim $w = m$).

The least-squares formulation (eqn. 2) gives the minimum-variance estimate if the measurement noise is Gaussian and has known covariance. However, in practice, it is unlikely that the noise statistics will be known in advance, and if measurements contain gross errors, such as large systematic errors, reversed sign of measurements or zero readings, the least-squares estimator will give poor results. Geometrically the state estimate achieved in eqn. 2 represents a point in the space of feasible solutions for which the sum of squares of distances between it and measurement hyperplanes is minimal. Thus, a gross measurement error will have an exaggerated effect on the state estimate, since the corresponding measurement hyperplane will be distant from the true solution point. Various techniques have addressed this problem [1, 2] but rely on a complete least-squares estimation followed by a filtering procedure.

The formulation (eqn. 3) of the state-estimation problem adopted in this paper minimises the sum of moduli of measurement residuals, which represents geometrically the sum of distances from measurement hyperplanes. The solution point is defined by the intersection of n hyperplanes with the smallest measurement noise. This gives rise to a potential for the total rejection of bad data from the measurement set provided that the number of gross errors does not exceed $m - n$. The triangle inequality in Euclidean space ensures that the deviant measurement hyperplane, unlike the least-squares formulation, does not influence the solution point, since this would increase distances to the remaining $n - 1$ hyperplanes. The penalty incurred in achieving the rejection of bad data is that n measurements spanning the solution are included with their associated measurement noise. However, in the water distribution systems field, this has only a minimal effect on the estimates computed.

3 Formulation of the problem

The proposed solution of the state-estimation problem (eqn. 3) is based on the Newton method, which has been widely used in water network computations [5, 6, 7].

Expanding $g(x)$ by an initial guess of the state vector x^0 using a first-order Taylor series and defining $z^0 = g(x^0)$, we have

$$z = z^0 + \Delta z \tag{4}$$

$$g(x) = g(x^0) + J(x^0)\,\Delta x \tag{5}$$

Eqn. 3 can be therefore expressed as

$$\min_{\Delta x} C = w^T | \Delta z - J(x^0)\,\Delta x | \tag{6}$$

where $J(x^0)$ is an $m \times m$ Jacobian matrix evaluated at x^0.

Because a direct solution of eqn. 6 is computationally inconvenient, artifical variables r and s can be introduced with the following definitions:

$$s_i = \begin{cases} \Delta z_i - J_i(x^0)\,\Delta x & \text{if} \quad \Delta z_i \geqslant J_i(x^0)\,\Delta x \\ 0 & \quad\quad \text{otherwise} \end{cases} \tag{7}$$

$$r_i = \begin{cases} -(\Delta z_i - J_i(x^0)\,\Delta x) & \text{if} \quad \Delta z_i < J_i(x^0)\,\Delta x \\ 0 & \quad\quad \text{otherwise} \end{cases} \tag{8}$$

for $i = 1, 2, \ldots, m$.

Eqn. 6 can then be represented as

$$\min_{r, s, \Delta x} C = W^T \begin{vmatrix} \Delta x \\ r \\ s \end{vmatrix} \tag{9}$$

subject to

$$\Delta z - J(x^0)\,\Delta x + r - s = 0$$

where vector $W^T = [0^T : w^T : w^T]^T$.

Solution of eqn. 9 yields Δx, and, by repetition of the procedure, successive approximations of the state vector are given by

$$x^{k+1} = x^k + \alpha\,\Delta x^k \quad k = 0, 1, \ldots. \tag{10}$$

with coefficient α chosen in such a way as to produce a norm-reducing process.

4 Solution technique

The optimisation problem of eqn. 9 may be written in standard linear programming form:

$$\min_y W^T \cdot y \tag{11}$$

subject to

$$Ay = b$$

where $A = [J(x^k) : 1 : -1]$ $(k = 0, 1, \ldots)$, $b = \Delta z^k + J(x^k)d$, $y^T = [(\Delta x^k + d)^T : r^T : s^T]^T$ and d is a constant vector with elements sufficiently large to guarantee $\Delta x_i^k + d_i \geqslant 0$ $(i = 1, \ldots, n)$.

A sparse, revised Simplex method has been used to solve eqn. 11, and the solution at the kth Newton iteration may be expressed as

$$y_B = (B^k)^{-1} b^*$$

$$y_N = 0 \tag{12}$$

where y_B and y_N are, respectively, basic and nonbasic variables in the LP solution.

In order to fully exploit the sparsity of matrix A in the linear program, the solution algorithm should be designed to minimise the number of new elements created during the factorisation at each iteration. It is now well recognised [8] that the elimination form of matrix inverse factorisation leads to better retention of sparsity than the product form. By the introduction of further interchanges of rows and columns, Reid [9] has achieved an algorithm which avoids, whenever possible, pivotal operations with consequent fill-in. This algorithm, in the form of Harwell subroutine LA05A, has been used for handling the basis inverse. As the solution of eqn. 11 is attempted in each Newton iteration, it is crucial to achieve the best computational efficiency for the algorithm. In the standard case the Simplex algorithm begins with a unit matrix as an initial feasible basis. However, by exploiting the special features of the water-system state-estimation problem, it is possible to avoid such a flat start and to construct a feasible basis which needs very few Simplex iterations to produce an optimal solution.

This procedure can be summarised as follows:

(*a*) construct matrix F consisting of the columns of the Jacobian matrix corresponding to the basic state variables in the previous optimal solution

(*b*) factorise matrix F into two factors F^A and F^R where F^A consists of the rows of F which represent measurements with zero slack variables in the optimal solution (dim $F^A \leqslant n$), and F^R consists of the remaining rows of F.

$$F^T = \left[\begin{array}{c} F^A \\ \hline F^R \end{array}\right]$$

and the corresponding RHS vector is

$$b = \left[\begin{array}{c} b^A \\ \hline b^R \end{array}\right]$$

(*c*) solve the reduced set of linear equations $F^A y^A = b^A$.

(*d*) form an initial feasible basis for the Simplex algorithm following the scheme:

If:

$$y_j^A \geqslant 0 \text{ for all } j = 1, \ldots, \dim F^A$$

then

$$B^1 = \left[\begin{array}{c|c} F^A & 0 \\ \hline F^R & U \end{array}\right]$$

$$b = \left[\begin{array}{c} b^A \\ \hline b^R - F^R y^A \end{array}\right]$$

and U is a diagonal matrix with elements

$$u_{ii} = \operatorname{sgn}(b_i^R - F_i^R y^A), \quad i = 1, \ldots, (m - \dim F^A)$$

else

B^1 is chosen in conventional way

In online applications of the state estimator, the above procedure can also be utilised in the first iteration of the Newton process exploiting the availability of the state estimate from the previous time step.

5 Test results

The performance of the proposed method for water-system state estimation has been tested on the two networks presented in Figs. 1 and 2. In the absence of published test network data for water distribution systems, Tables 1 and 4 contain a complete definition of network parameters to allow future comparison of results. The measurement data

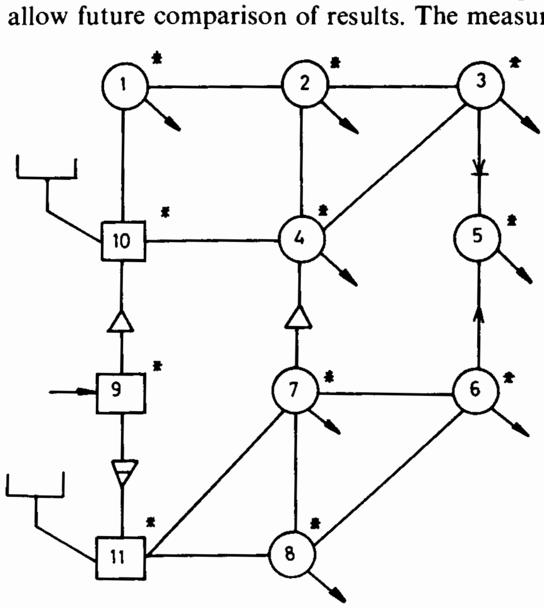

Fig. 1 *11-node network*

- load or inflow
- load node
- fixed-head node
- parabolic pump
- fixed-head pump
- one-way value
- pressure reducing valve
- measurement point

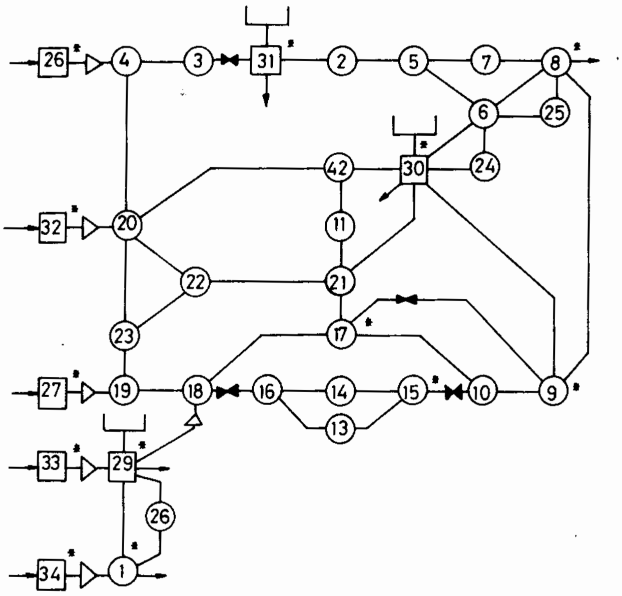

Fig. 2 *34-node network*

- load or inflow
- load node
- fixed-head node
- parabolic pump
- valve
- measurement point

was obtained by exact network simulation with the subsequent addition of a small random noise component of a magnitude proportional to the measurement value (0.05%). In order to achieve sufficient measurement redundancy, the set of available measurements was increased by the introduction of pseudomeasurements representing a mass balance in each node with a zero load. The effect of 'bad data' measurements was simulated by modificaton of measured values to zero, upper or lower limits, or by changing their sign. These modifications are

Table 1: Data for 11-node network

Links	Link parameters				Nodes	Measurements/pseudomeasurements			
Pipe $N_1 - N_2$	Length, m	Diameter, m	CHW	—		Head, mAq	Load, m^3/s	Inflow, m^3/s	Flow, m^3/s
10– 1	914.4	0.4064	100		1		−0.02832		
2– 1	914.4	0.3048	120		2	145.8	−0.02832		
3– 2	609.6	0.2540	110		3		−0.02832		
4– 2	609.6	0.3048	115		4		−0.02832		
10– 4	609.6	0.3048	110		5		−0.05664		
4– 3	609.6	0.2540	100		6	93.2	−0.02832		
6– 7	609.6	0.2540	110		7		−0.02832		
8– 7	609.6	0.2032	100		8		−0.05664		
11– 8	609.6	0.3048	110		9	60.9	0.0P	0.2832	
7–11	1219.2	0.2540	100		10	146.3	0.0P		
8– 6	609.6	0.2540	120		11	115.8	0.0P		
					7–4				0.1545*

Parabolic pump	a, s^2/m^5	b, s/m^2	c, m	
7–4	−190.1	0.0	60.96	
9–10	−1140.1	0.0	85.39	
One-way valve	**Length, m**	**Diameter, m**	**CHW**	
6–5	1219.2	0.2032	100	
Pressure valve	**Length, m**	**Diameter, m**	**CHW**	**Limit press., m**
3–5	1219.2	0.2032	110	109.72
Fixed head pump-m	**Length, m**	**Diameter, m**	**CHW**	**Head incr., mAq**
9–11	304.8	0.4064	65	65.52

* Measurement not used in example 1.4
P = pseudomeasurement
CHW = Hazen-Williams coefficient
$N_1 - N_2$ = sending and receiving nodes of the pipe

Table 2: 11-node-network erroneous measurements

Example	Type of measurement	Exact value	Erroneous value	Corrected value
1.1	load in node 7	−0.02832	0.0	−0.02832
1.2	load in node 7	−0.02832	0.0	−0.02812
	flow between node 7 and node 4	0.15450	0.14450	0.15447
1.3	load in node 7	−0.02832	0.0	−0.02890
	load in node 4	−0.02832	0.02832	−0.02848
	head in node 8	100.40	90.40	100.39
	head in node 9	60.97	90.97	60.97
1.4	load in node 7	−0.02832	0.0	−0.02775

Load units: m^3/s
Flow units: m^3/s
Head units: m Aq

Table 3: 11-node-system state estimates

State variable	Exact state	Example 1.1	Example 1.2	Example 1.3	Example 1.4
1	145.892	145.888	145.888	145.892	145.889
2	145.759	145.756	145.756	145.769	145.756
3	144.157	144.156	144.154	144.157	144.260
4	147.006	147.009	147.004	147.006	147.267
5	92.085	92.082	92.082	92.085	92.082
6	93.229	93.229	93.229	93.229	93.229
7	90.560	90.586	90.564	90.570	91.628
8	100.402	100.401	100.398	100.388	100.576
9*	60.964	60.960	60.960	60.969	60.960
10*	146.307	146.304	146.303	146.307	146.304
11*	115.781	115.770	115.770	115.781	115.770
12	0.2832	0.2832	0.2832	0.2832	0.2832
13	0.0	0.0	0.0	0.0	0.0
14	0.0	0.0	0.0	0.0	0.0

State variables
1–11: nodal heads (m Aq) at nodes 1–11
12–14: fixed-head modes in/out flows (m^3/s) at nodes 9–11
* Extended-state variables

intended to simulate typical malfunctions of some instrumentation or associated telemetry, which would give rise to a corrupted measurement set. A large number of tests have been conducted, but only representative samples are given here.

For the 11-node network with three fixed-head nodes (14 state variables) two sets of measurements were processed having redundancy ratios of 1.35 and 1.28.

Example 1.1 illustrates the effect of a loss of the measurement in node 7, which results in a zero reading instead of the correct load value: 0.02832 m^3/s. The estimator rejects bad data giving a residual 0.02832, which converts the load to -0.02832 m^3/s. Discrepancies in the remaining measurements ($m - n - 1$), which were also rejected in the estimation process, were small and represent the added Gaussian measurement noise.

In example 1.2 the measurement data of example 1.1 have been modified by introduction of a systematic gross error of flow measurement between nodes 7 and 4. The least absolute value estimator has successfully identified the errors and corrected the faulty measurements. The accuracy of the state estimate is also not noticeably affected.

The combined effect of a lost measurement of demand

Table 4: Data for 34-node network

Links	Link parameters			Nodes	Measurements/pseudomeasurements			
Pipe $N_1 - N_2$	Length, m	Diameter m	CHW		Head, m Aq	Load m^3/s	Inflow, m^3/s	Flow, m^3/s
4–3	606.6	0.4572	110	1	32.66	−0.0566		
20–4	454.2	0.4572	110	2	43.76	0.0P		
23–20	2782.8	0.2286	105	3	46.04*	0.0P		
23–19	304.8	0.3810	135	4	46.62	0.0P		
20–12	3383.3	0.3048	105	5	43.28*	0.0P		
20–22	1767.8	0.4572	110	6	43.02*	0.0P		
22–23	1015.0	0.3810	135	7	42.40*	0.0P		
19–18	1097.3	0.3810	135	8	42.13	−0.0750		
3–31	1930.9	0.4572	110	9	43.78*	0.0P		
31–2	3150.6	0.3048	100	10	47.95*	0.0P		
22–21	762.0	0.4572	110	11	44.67	0.0P		
18–17	914.4	0.2286	125	12	44.00*	0.0P		
18–16	823.0	0.3048	140	13	49.30*	0.0P		
16–14	411.5	0.1524	100	14	49.10*	0.0P		
14–15	701.0	0.2286	110	15	49.05	0.0P		
16–13	1072.9	0.2286	135	16	49.31*	0.0P		
13–15	864.1	0.1524	90	17	47.97	0.0P		
15–10	711.7	0.1524	90	18	49.33*	0.0P		
10–17	832.1	0.1524	90	19	49.06	0.0P		
17–9	2334.8	0.1524	100	20	46.62*	0.0P		
17–21	1969.4	0.2286	95	21	45.63*	0.0P		
21–11	777.2	0.2286	90	22	46.59	0.0P		
11–12	542.5	0.2286	90	23	48.37*	0.0P		
21–30	1600.2	0.4572	110	24		0.0P		
30–12	249.9	0.3048	105	25		0.0P		
2–5	1028.7	0.2286	110	26		0.0P		
30–24	443.7	0.2286	90	27	−15.24	0.0P	0.0723	
30–6	743.7	0.3810	100	28	−33.53	0.0P	0.0927	
30–9	931.1	0.2286	125	29	31.70	−0.0253	−0.0229	
9–10	2689.9	0.1524	100	30	43.59	−0.0421	−0.0522°	
5–7	326.1	0.1524	100	31	44.20	−0.0421	−0.0229°	
7–8	844.3	0.2286	110	32	−45.72	0.0P	0.0254	
5–6	1274.0	0.1524	100	33	−36.58	0.0P	0.0614	
6–8	1115.6	0.2286	90	34	−12.19	0.0P	0.1063	
6–25	615.6	0.3810	110	34–1				0.1063*
8–9	1406.7	0.1524	100	33–29				0.0614*
1–29	426.7	0.2540	100	27–19				0.0723*
1–26	2098.1	0.3556	100	32–20				0.0254*
25–8	500.0	0.3810	110	28–4				0.0927*
24–6	300.0	0.2286	90	29–18				0.0628*
26–29	1500.0	0.3556	100					

Parabolic pump	a, s^2/m^5	b, s/m^2	c m
28–4	−4921.8	0.0	122.44
32–20	−444.4	−385.4	102.42
27–19	−812.3	89.7	62.03
29–18	−318.6	0.0	18.89
34–1	−812.3	89.7	44.50
33–29	−4162.6	138.4	75.47

* Measurements used only in examples 2.1 and 2.2
° Measurements used only in examples 2.3 and 2.4
P = pseudomeasurements

at node 7, reversed sign of demand measurement in node 4 and two gross systematic errors of head measurements in nodes 8 and 9 is illustrated in example 1.3. Similarly, in this situation, the estimator was able to identify and correct the gross errors; however, the reduction in the number of nonperturbed measurements resulted in less effective noise filtering.

The relationship between measurement redundancy and computational accuracy of the state estimate has also been investigated. The measurement data of example 1.4 are directly comparable with example 1.1, being reduced only by the reading marked with an asterisk in Table 1. With a smaller number of available measurements, the estimator also rejects the gross error (zero reading), but the value of the correction is influenced more by the Gaussian measurement noise. This is also apparent from the state estimates given in Table 3. The lower limit for measurement

Table 5: 34-node-network erroneous measurements

Example	Type of measurement	Exact value	Erroneous value	Corrected value
2.1	head in node 22	46.59	42.59	46.49
	load in node 8	−0.0750	−0.0250	−0.0753
2.2	head in node 22	46.59	42.59	46.59
	head in node 29	31.69	35.70	31.70
	head in node 30	43.58	48.58	43.59
	load in node 8	−0.0750	−0.0250	−0.0753
2.3	head in node 22	46.59	42.59	46.58
	load in node 8	−0.0750	−0.0250	−0.0754
2.4	head in node 22	46.59	42.59	46.59
	head in node 29	31.69	35.70	31.70
	head in node 30	43.58	48.59	43.59
	load in node 8	−0.0750	−0.0250	−0.0754

Load units: m^3/s
Flow units: m^3/s
Head units m Aq

Table 6: 34-node-system state estimates

State variable	Exact state	Example 2.1	Example 2.2	Example 2.3	Example 2.4
1	32.638	32.655	32.663	32.646	32.662
2	43.749	43.756	43.759	43.753	43.759
3	46.041	46.042	46.043	46.041	46.042
4	46.618	46.621	46.623	46.620	46.622
5	43.265	43.272	43.275	43.274	43.277
6	43.024	43.025	43.026	43.025	43.026
7	42.402	42.399	42.398	42.402	42.400
8	42.130	42.133	42.134	42.132	42.134
9	43.798	43.790	43.782	43.806	43.784
10	47.950	47.949	47.948	47.946	47.948
11	44.664	44.668	44.670	44.667	44.670
12	44.004	44.004	44.004	44.004	44.005
13	49.274	49.284	49.294	49.277	49.284
14	49.099	49.100	49.101	49.117	49.107
15	49.057	49.058	49.059	49.057	49.059
16	49.298	49.316	49.315	49.304	49.311
17	47.970	47.970	47.970	47.971	47.970
18	49.338	49.337	49.336	49.346	49.338
19	49.029	49.029	49.030	49.036	49.035
20	46.618	46.621	46.622	46.619	46.622
21	45.623	45.626	45.628	45.620	45.628
22	46.588	46.591	46.593	45.581	46.594
23	48.379	48.376	48.375	48.381	48.379
24	43.249	43.251	43.252	43.250	43.252
25	42.532	42.534	42.534	42.533	42.534
26	32.086	32.096	32.101	32.091	32.102
27*	−15.233	−15.238	−15.239	−15.235	−15.239
28*	−33.521	−33.522	−33.522	−33.522	−33.528
29*	31.692	31.697	31.700	31.694	31.701
30*	43.582	43.585	43.587	43.584	43.586
31*	44.188	44.193	44.195	44.191	44.195
32*	−45.710	−45.716	−45.719	−45.714	−45.719
33*	−36.572	−36.574	−36.574	−36.574	−36.576
34*	−12.184	−12.189	−12.191	−12.187	−12.191
35	0.07229	0.07230	0.07230	0.07229	0.07230
36	0.09267	0.09269	0.09270	0.09268	0.09270
37	−0.02290	−0.02290	−0.02290	−0.02290	−0.02290
38	−0.05183	−0.05201	−0.05188	−0.05220	−0.05220
39	−0.03910	−0.03910	−0.03910	−0.03910	−0.03910
40	0.02538	0.02539	0.02540	0.02539	0.02540
41	0.06134	0.06138	0.06140	0.06136	0.06139
42	0.10604	0.10621	0.10627	0.1061	0.10627

* Extended state variables
State variables
1–34: nodal heads (m Aq) at nodes 1–34
35–42: fixed-head nodes in/out flows (m^3/s) at nodes 27–34

redundancy, which assures an acceptable level of noise filtering, was found to be mainly a function of measurement configuration and measurement noise level. If these parameters are kept constant, the introduction of additional measurements results in only a slight improvement in computational accuracy of the estimator.

The realistic 34-node network of Fig. 2 with eight fixed-head nodes (42 state variables) was tested for an existing and extended measurement set with corresponding redundancy ratios 1.4 and 1.81. The salient feature of the measurement configuraton in this network is the uneven distribution of metering points. In consequence, the state-estimation problem is numerically less well defined. The gross errors have been introduced in the form of systematic errors, which are, in practice, the most difficult to identify by the common preprocessing procedures such as limit or consistency checking. Comparison of the results of examples 2.1 and 2.2, in which higher measurement redundancy has been used, with the results of examples 2.3 and 2.4, indicates that the lower redundancy is still sufficient for estimation purposes. Thus, an increased number of measurements contributes mainly to an improved reliability of the measurement set. This confirms the results of example 1.4.

The state estimator generally converges to the solution in 3–4 iterations of modified Newton process, but an increase in the number of erroneous measurements and a decrease of redundancy ratio may produce one or two additional iterations. Computation time and the number of Newton iterations required for solution are presented in Table 7.

The algorithm has been coded in Fortran 77 and implemented on a Perkin-Elmer 3220 minicomputer with 32-bit

Table 7: Computer time requirements

Example	Iterations	Time, s
Exact solution	3	2.62(2.43)*
1.1	3	2.58(2.39)
1.2	3	2.65(2.50)
1.3	3	3.98(3.51)
1.4	3	3.07(2.72)
Exact solution	3	13.21(8.42)
2.1	3	15.95(8.37)
2.2	3	15.96(7.80)
2.3	3	12.83(8.20)
2.4	3	16.32(9.47)

* Times in parentheses refer to a version incorporating an initial estimate of the basic variables

word length and hardware floating-point arithmetic. In its present version, the program occupies 28 k words (112 k bytes) of memory.

6 Conclusions

A technique for improvement of measurement accuracy in the presence of large non-Gaussian random disturbances has been presented. The algorithm has been shown to be suitable for online implementation with only very modest computational requirements. The state estimate for the water distribution network has been achieved by a minimisation of the sum of moduli of measurement errors rather than the more conventional least-squares function. The application of sparse matrix techniques for solution of the resulting linear program and the introduction of a restarting algorithm for successive state estimates has produced a computationally efficient method.

The performance of the algorithm has been tested on many different types of network, two of which have been presented: one designed to highlight any problems with convergence and the second a comprehensive representation of an actual water distribution system. The results obtained indicate that the algorithm is robust and capable of correctly rejecting gross measurement errors. A further improvement in noise filtering properties can be obtained by the inclusion of additional network topology information in the form of pseudomeasurements.

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8 References

1 BROUSSOLLE, F.: 'State estimation in power systems: detecting bad data through the sparse inverse method', *IEEE Trans.*, 1978, **PAS-97**, pp. 678–682

2 PREWETT, J.N., FARMER, E.D., LAING, W.D., and JERVIS, P.: 'Studies of a state estimation procedure for a power system and its online performance' *in* 'Online operation and optimization of transmission and distribution systems'. IEE Conf. Report Series 140, 1976

3 IRVING, M.R., OWEN, R.C., and STERLING, M.J.H.: 'Power-system state estimation using linear programming', *Proc. IEE*, 1978, 125, pp. 879–885

4 PIOTROWSKI, J.: Report on National Conference on Water-Distribution Systems, Ustron, 1978, (in Polish)

5 RAO, H.S., BREE, D.W., and BENZVI, R.: 'Extended period simulation of water distribution networks'. Final report by Systems Control Inc. Document PB-230 148, Palo Alto, CA, Feb. 1974

6 COULBECK, B.: 'Optimisation and modelling techniques in dynamic control of water distribution systems'. Ph.D. Thesis, University of Sheffield, July 1977

7 RAHAL, C.M., STERLING, M.J.H., and COULBECK, B.: 'Parameter tuning for simulation models of water distribution networks'. *Proc. Instn. Civil Eng.*, 1980, **69**, (2), pp. 751–762

8 MARKOWITZ, H.M.: 'The elimination form of the inverse and its applications to linear programming', *Management Sci.*, 1957, **3**, pp. 255–269

9 REID, J.K.: 'A sparsity-exploiting variant of the Bartel-Golub decomposition for linear programming bases'. Report CSS 20, AERE Harwell, 1975