SET POINT AND IDENTIFIABILITY IN THE CLOSED LOOP WITH A MINIMUM-VARIANCE CONTROLLER

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Abstract. The meaning of nonzero set points for the identifiability of the ARMAX model parameters in a closed-loop system with a minimum-variance controller is explained and discussed. It is pointed out that, in the case of a zero set point and an ARMAX model resulting from the discretization of a continuous-time plant, the model parameters cannot be identified in a closed-loop system with a minimum-variance controller. It is also shown that in the case of appropriately varying set point or the existence of a delay in the plant, the identifiability conditions of the parameters can be fulfilled.

Keywords. Identifiability; self-tuning controllers; minimum variance control; parameter estimation; closed-loop systems.

1. INTRODUCTION

The identification of varying model parameters is the essential point of adaptive control systems, and especially of self-tuning controllers. The self-tuning controller is based on the so-called "separation principle", i.e. on the idea of separating the design of the controller from the identification of unknown parameters. The unknown parameters are identified on-line in a closed-loop system by using a recursive estimation method. Two different types of self-tuning controllers are known: indirect (explicit) controllers and direct (implicit) ones. In an indirect self-tuning scheme the parameters of the transfer function are explicitly estimated, and then the parameters of a controller are calculated indirectly on the basis of the estimated parameters.

There are large numbers of possible types of controllers and recursive estimation algorithms which can be used as a basis for self- tuners. They should be designed to work in a wide spectrum of conditions defined by the behaviour of disturbances, set-point values and system parameters. Although the theory of self-tuning control is quite well established (Åström and Wittenmark, 1989; Isermann *et al.*, 1992; Kosut *et al.*, 1987), the theoretical results concerning e.g. stability, optimality, order estimation, time-varying parameters and the consistency of plant parameter estimates are only valid under special conditions and for selected groups of self-tuners. Widespread assumptions are: constant parameters, standard recursive estimation methods without data forgetting, and zero set points. In practice, parameters do change, the forgetting mode is used in estimation, and a zero set point is rarely present.

A sufficiently rich excitation of the system plays an essential role for estimation algorithms. This is expressed by the so-called "persistent excitation" (PE) condition. In closed-loop systems the PE condition can be violated even in the case of sufficiently rich exciting noises, which can produce problems with identifiability (Gustavsson *et al.*, 1977) and convergence.

The present paper concerns minimum-variance self- tuning control. In particular, attention is focused on situations in which the identifiability condition can, or cannot, be fulfilled in a closed-loop system with a minimum-variance controller. It is noted that in the frequently occurring case in which a discrete-time plant results from the discretization of a continuous-time plant described by a rational proper transfer function with zero-order hold, the identifiability condition cannot be fulfilled in a closed-loop system with zero set point and any exciting noise. It is also noted in the case of appropriately varying set point or of the existence of some delay in the continuous-time plant, as well as in the case of applying an incremental model, that the identifiability condition in the closed-loop system can be fulfilled for sufficiently rich exciting noise.

2. MINIMUM VARIANCE SELF–TUNING CONTROL

System models used in self-tuning control are assumed to be local linearizations of typically nonlinear systems. If $U(i)$ denotes an input, and $Y(i)$ an output signal, then

$$Y(i) = \bar{y} + y(i) \quad (1)$$

$$U(i) = \bar{u} + u(i) \quad (2)$$

consist of nonzero mean levels, $\bar{u}$ and $\bar{y}$, and the perturbations, $u(i)$ and $y(i)$, around them, where

$$\bar{y} = \varphi(\bar{u}) \quad (3)$$

is a steady-state characteristic of the (usually nonlinear) plant to be controlled.

It is very often assumed that the linear dynamic model for perturbations is in the ARMAX form

$$A(z^{-1})y(i) = B(z^{-1})u(i-d) + C(z^{-1})v(i) \quad (4)$$

where

$$y(i) = z^{-1}y(i+1) \quad (5)$$

$$v(i) \sim n.i.d(0, \sigma^2) \text{ white noise} \quad (6)$$

and

$$A(z^{-1}) = 1 + a_1 z^{-1} + ... + a_n z^{-n} \quad (7)$$

$$B(z^{-1}) = b_0 + b_1 z^{-1} + ... + b_n z^{-n} \quad (8)$$

$$C(z^{-1}) = 1 + c_1 z^{-1} + ... + c_n z^{-n}. \quad (9)$$

Another kind of model (4) is

$$A(z^{-1})\Delta y(i) = B(z^{-1})\Delta u(i-d) + C(z^{-1})v(i) \quad (10)$$

written in increments

$$\Delta u(i) = u(i) - u(i-1) = U(i) - U(i-1) \quad (11)$$

$$\Delta y(i) = y(i) - y(i-1) = Y(i) - Y(i-1). \quad (12)$$

Model (10) allows the incorporation of unknown constant terms of the input and output, and thus goes very well with the model of (1)-(3).

For real-time identification, recursive parameter-estimation methods have been developed for the above linear processes with time-invariant parameters. In the case of self-tuning control, the plant parameters vary in time and algorithms such as the recursive extended least-squares (RELS) estimation algorithm in the weighted version can be applied. It is described by the formulas

$$\hat{\Theta}(i+1) = \hat{\Theta}(i) + k(i)[y(i+1) - \varphi'(i+1)\hat{\Theta}(i)] \quad (13)$$

$$k(i) = \mu(i+1)P(i)\varphi(i+1) \quad (14)$$

$$\mu(i+1) = [\lambda + \varphi'(i+1)P(i)\varphi(i+1)]^{-1} \quad (15)$$

$$P(i+1) = \lambda^{-1}\left[P(i)\frac{P(i)\varphi(i+1)\varphi'(i+1)P(i)}{\lambda + \varphi'(i+1)P(i)\varphi(i+1)}\right] \quad (16)$$

where $\lambda < 1$ is a forgetting parameter and

$$\Theta' = [a_1, ..., a_n, b_0, ..., b_n, c_1, ..., c_n] \quad (17)$$

$$\varphi'(i+1) = [-y(i), ..., -y(i-n+1), u(i-d+1), ..., u(i-d-n+1), \varepsilon(i), ..., \varepsilon(i-n+1)] \quad (18)$$

$$\varepsilon(i) = y(i) - \varphi'(i)\hat{\Theta}(i). \quad (19)$$

That is, in (18), in the place of $v(i)$, its estimate $\varepsilon(i)$, determined by (19), is utilised.

Using the notations introduced above, the model (4) can be described by the approximate formula

$$y(i+1) = \varphi'(i+1)\Theta + \varepsilon(i+1). \quad (20)$$

After performing transformations which can be found in the standard textbooks on RLS estimation, from (16)

$$P^{-1}(i+1) = \lambda P^{-1}(i) + \varphi(i+1)\varphi'(i+1) \quad (21)$$

and also

$$P^{-1}(i) = \underline{\varphi}(i)\underline{\varphi}'(i) \quad (22)$$

$$\underline{\varphi}'(i) = \begin{bmatrix} \sqrt{\lambda}^{i} & \varphi'(0) \\ \sqrt{\lambda}^{i-1} & \varphi'(1) \\ \cdots\cdots & \cdots\cdots \\ \sqrt{\lambda}^{1} & \varphi'(i-1) \\ \sqrt{\lambda}^{0} & \varphi'(i) \end{bmatrix}. \quad (23)$$

Attention is restricted here to the minimum-variance controller which minimises the performance index

$$I = E\{y(i+d) - w(i)\}^2 = E\{e(i+d)\}^2 \quad (24)$$

where $w(i)$ and $e(i+d)$ are the set point at time i and the "control error at time $i+d$", respectively. If the model (4) is minimum-phase, the minimum-variance controller is given by

$$B(z^{-1})F(z^{-1})u(i) = C(z^{-1})w(i) - G(z^{-1})y(i) \quad (25)$$

where $F(z^{-1})$ and $G(z^{-1})$ result from solving the Diophantine equation

$$A(z^{-1})F(z^{-1}) + z^{-d}G(z^{-1}) = C(z^{-1}). \quad (26)$$

If the model parameters of (4) are known, then in a closed-loop system with the controller (25) the control error becomes a moving average of order $d-1$:

$$e(i) = F(z^{-1})v(i). \quad (27)$$

The indirect self-tuning controller of Åström and Wittenmark (1989) is described by the following algorithm:

- estimate the coefficients of $A(z^{-1})$, $B(z^{-1})$ and $C(z^{-1})$ using e.g. the weighted version of the RELS algorithm;
- solve the Diophantine equation using $\hat{A}(z^{-1})$, $\hat{B}(z^{-1})$ and $\hat{C}(z^{-1})$ and determine the controller equation (25);
- calculate the control signal from (25);
- repeat the above steps for each sampling period.

In the place of RELS, an appropriate version of the RPEM (recursive prediction error method) can be used (Ljung and Söderström, 1977).

A minimum-variance controller for system (10) has the form

$$B(z^{-1})F(z^{-1})\Delta u(i) = C(z^{-1})w(i) - G(z^{-1})y(i) \quad (28)$$

and can be easily found by solving a modified Diophantine equation

$$(1-z^{-1})A(z^{-1})F(z^{-1}) + z^{-d}G(z^{-1}) = C(z^{-1}) \quad (29)$$

for $F(z^{-1})$ and $G(z^{-1})$. As a result, a controller with an integral action is obtained.

It should be stressed that the convergence of the controller parameters does not necessarily mean the consistency of the plant parameters for an indirect self-tuning controller. In fact, very often the plant parameters in a closed-loop system cannot be identified even if the exciting noise is sufficiently rich. This problem will be discussed in the next section.

3. SOME QUESTIONS OF IDENTIFIABILITY

3.1. The case of a continuous-time plant without delay

Consider the case when the polynomials $A(z^{-1})$ and $B(z^{-1})$ and the delay d in the ARMAX model (4) result from the discrete-time description of the system composed of a sampler, ZOH and continuous-time plant (described by the rational proper transfer function $G(s)$) in series. In other words the discrete-time dynamic model (4) (without the part concerning the disturbance) results from the discretization with ZOH of the continuous-time plant $G(s)$ without delay. This case appears very frequently in applications.

It is known that for this kind of discrete-time system see e.g. (Åström and Wittenmark, 1989), independently of the value of the sampling interval, there is

$$d = 1, \quad b_n = 0 \quad (30)$$

and

$$F(z^{-1}) = 1$$
$$G(z^{-1}) = g_0 + g_1 z^{-1} + \ldots + g_{n-1}z^{-n+1} \quad (31)$$

i.e. the polynomials $B(z^{-1})$ and $G(z^{-1})$ have the same degree.

It will be shown that, in this case, the varying set point plays an essential role in the model parameter's identifiability.

In reality, if $w(i) = 0$ then the controller equation (25) determines some linear dependence between the first $2n$ columns of matrix (23), determined also by (18). This, taking into account (22), means that matrix $P^{-1}(i)$ is singular, and the inverse matrix $P(i)$ does not exist. Further, the matrix calculated by means of formula (16) does not determine $P(i)$ (the formula (16) was derived under the assumption that $P^{-1}(i)$ is non-singular). As a result, the parameter estimates calculated by using the weighted RELS algorithm are incorrect.

Strictly speaking, the statements in the paragraph above are valid under constant controller parameter values. It should also be noticed, however, that in the periods in which the controller parameters vary as the result of new estimates, the matrix $P^{-1}(i)$ can be non-singular. In these periods the estimation algorithm can work. Nevertheless, for constant plant parameters, when the controller parameter estimates become constant, the matrix $P^{-1}(i)$ becomes singular and the algorithm stops working. As the result a drift of the plant parameter estimates is observed, and the system can become unstable.

On the other hand, if the set point $w(i) \neq 0$ appears in formula (25) then this formula does not determine a linear dependence between the columns of matrix (23). At the first look, it seems that then the identifiability condition can be fulfilled. However, the simulations performed do not confirm this view. To justify this observation, consider equations (4) and (25) in the incremental form i.e. substitute in them the variables $\Delta u(i)$

$\Delta y(i)$ and $(1-z^{-1})v(i)$ ($\Delta u(i) = u(i) - u(i-1)$, $\Delta y(i) = y(i) - y(i-1)$) in the place of $u(i), y(i)$ and $v(i)$, respectively. Then, from replacing the variables $u(i)$ and $y(i)$ by $\Delta u(i)$ and $\Delta y(i)$ in the RELS algorithm, it results that for $w = const \neq 0$ the estimation has similar properties as for $w = 0$.

3.2. The case of a continuous-time plant with delay

If the continuous-time plant contains a delay, then for an appropriately small sampling period $d > 1$. It can be noted that in this situation as well, formula (25), even for $w(i) = 0$, does not determine a linear dependence between the columns of matrix (23), since then in (25) some variables appear which do not appear in (23). It seems, that in this case the model parameters can be identified if the system excitation is sufficiently rich. The system described by the incremental model (10) with the control algorithm (28) has similar properties.

4. THE LOW-EXCITATION PROBLEM

Excitation is necessary for parameter estimation. If a persistently exciting signal is present, then the estimates can be convergent. Many authors, both theoreticians and practitioners, have raised the problem of poor or low excitation. However, it seems that the two cases should be distinguished. In the first, commonly appearing case, the simulations are performed numerically, with floating-point arithmetic and high accuracy of calculations. In the second, rather infrequent, case the experiments are performed on a real physical system, where the quantization of the A /D converters plays an essential role. The following discussion is concerned mainly with the first case; however, after appropriate interpretation it can also be related to the second one.

First it is necessary to point out a false interpretation of the notion of low excitation which simply assumes a small value of the variance σ in the model (4) or (10). Indeed, if the set point $w(i) = 0$ then the estimation equations $(13)-(16)$ become independent of σ if the normalised values $P(i) = P_0(i)//\sigma^2$ and $\varphi(i) = \varphi_0(i)\sigma$ are substituted in them. In other words "smallness" is a relative notion, and in the case $w(i) = 0$ there is no reference level.

When both σ and $w(i)$ are equal to zero then the system is completely "frozen", and no changes occur whatever the controller parameters, and despite nonstationarity of the plant parameters. Simulations show that in the case $w(i) = c1(i)$ and under the assumption (30), for relatively small values of λ the estimated parameters start to drift after some period, and the system can become unstable. This results in large perturbations or "bursting" in the process variables. The rich excitation caused by bursting usually results in improved parameter estimation, so it is self-correcting. For given $\lambda < 1$, the period after which the estimates start to drift increases when the quotient σ/c decreases.

5. EXAMPLE

Consider the ARMAX model (4) described by

$$\begin{aligned}(1 - 1.6z^{-1} + 0.8z^{-2})y(i) = \\ = z^{-1}k(3 - 1.5z^{-1})u(i) + v(i)\end{aligned} \qquad (32)$$

i.e. $a_1 = -1.6$, $a_2 = 0.8$, $d = 1$, $b_0 = 3$, $b_1 = -1.5$ and k is a coefficient which will be varied at the time of simulation.

The controller equation takes the form

$$k(b_0 + b_1 z^{-1})u(i) = w(i) + (a_1 + a_2 z^{-1})y(i). \qquad (33)$$

Some tests were performed on this system using the SIMULINK ver. 1.3c package. Many experiments were performed, and the results of four of them are shown in Figs 1 – 4, respectively.

The results shown in Figs 1 and 2 concern the case when $k = 1$, and the noise $v(i)$ is $N(0, 0.01)$ in the periods [0,90) and [260,650], while in the period [90,260), v(i)=0. The experiments were performed for $\lambda = 0.985$. In Fig. 1 it is seen that for $w = 0$ the parameter estimates already begin drifting in the period [0,90). Then in [90,260) they are frozen, and in [260,650] their drifts become very large.

The results of Fig. 2, concerning the case of a stepwise change of the set point (i.e. $w = c1(t)$, $c = 1$) are completely different. For $10 < t < 350$ the parameter estimates are accurate; for $t > 350$ some insignificant drifts appear, in which the estimate errors are slowly increasing over time. It was noticed that for greater σ the drifts appear erlier in time and are faster, while for greater c they are later and slower, i.e. the estimate plots depend on the quotient σ/c. Notice that in this experiment the parameter λ was very close to 1. For somewhat smaller λ, say $\lambda = 0.97$, the initial period of accurate estimates appearing after a stepwise change of w is significantly shorter. It should be stressed that for $\lambda = 1$ the essential estimate drifts were not observed over a long period of time, even for $w = 0$. This can be explained by the fact that in this case, the initial non-singular values of the matrix P are not forgotten.

The results shown in Figs 3 and 4 concern the cases when parameter k is linearly decreasing in the interval [90,260) from $k = 1$ to $k = 0.5$, while in [0,90) $k = 1$ and in [260,650] $k = 0.5$. For these experiments $\lambda = 0.9$; owing to this it was possible to estimate the varying parameters kb_0 and kb_1. In the case shown in Fig. 3 the system has zero noise and is excited by a stepwise change of the set point w. The estimates are accurate in the periods of constant parameters, while in [90,260) they have some errors.

In the case shown in Fig. 4, in the intervals [0,90) and [260,650] the noise with $N(0, 0.01)$ additionally appears. It is seen that in the intervals when the noise appears the estimates are worse. They are somewhat improved directly after a stepwise change of the set point w, and then their accuracy is rapidly decreased by the appearance of noise, up to the next stepwise change of w. In the noise–free period the estimates in Figs 3 and 4 are mutually close to each other.

6. CONCLUSIONS

In many papers concerning self-tuning controllers and adaptive control systems, the research has been performed under the assumption of a zero set point. (Sripada and Fischer, 1987; Niederli/nski and Mo/sci/nski, 1992)

In the present paper it is shown, that in the case of a constant set point and the ARMAX models with $d = 1$ and $b_n = 0$ their parameters cannot be identified in a closed-loop system with a minimum-variance controller. In practice, these models appear frequently. They correspond to the discrete-time plants resulting from the discretization of a continuous-time plant described by a rational, proper transfer function with zero-order hold.

It is also pointed out that an appropriately varying set point can have an essential influence on identifiability of the plant parameters. The varying set point, however, does not frequently appear in practice. On the other hand, the rectangular variation of the set point is frequently used for testing the adaptive systems. Therefore, it should be remembered that this kind of testing can change the system performance.

The identifiability condition can also be fulfilled even for a zero set point in the case where a delay exists in the continuous-time plant, or in the case where an incremental model (10) is applied with its corresponding controller (28).

From the simulation experiments it is seen that after a stepwise change of the set point w there exists a period in which the parameter estimates remain accurate, despite the appearance of noise. Later on, the noise causes a slow drift of the estimates. The periods of accurate values of the parameter estimates appearing directly after a stepwise change of the set point from 0 to c are shorter if the quotient σ/c is larger. Similarly, even a small decrease of the parameter λ causes a significant shortening of this period. At the time of the appearance of noise, parameter identification is possible only for λ very close to 1.

Finally, it should be stressed that when a closed-loop stabilising system works well, i.e. when the disturbances are well compensated by the control u, giving almost constant output y, then the measurement of the pair (y, u) is not sufficient for identification. In this case the additional measurement of disturbance can improve the conditions of identification.

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