A Qualitative Representation of Evolving Spatial Entities in Two-dimensional Topological Spaces

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Abstract. This paper identifies a spatio-temporal model that represents the evolution of 2-dimensional spatial entities. The research proposes a qualitative reasoning approach for modelling time in Geographical Information Systems (GIS). Spatio-temporal relationships are inferred by combining properties of measured time (chronological sequences) and historical time (ordered events) to geometrical and topological representations of space. We identify a minimum set of spatial evolution relationships involving one or many inter-related geographic entities and present their semantics.

1. Introduction

The search for a better understanding of natural and anthropic phenomena is one of the major objectives of natural and human sciences. A wide range of applications in both the sciences and planning disciplines have developed models to explain and simulate real-world evolution. The validation and efficiency of these models are highly dependent on data quality and consistency in both space and time. This has led to a need for an integrated representation of geographical and historical data which corresponds as closely as possible to the way the real-world changes. Although various conceptual models combining space and time have been proposed (Langran 1992, Cheylan 1993, Peuquet 1994, Frank 1994, Worboys 1994, Galton 1995), research on temporal GISs (TGIS) is still in an early stage of development. The development of TGISs requires a formal foundation comparable to the work realised for spatial models. A comprehensive framework and mathematical models of spatio-temporal relationships are needed for the development of flexible and efficient geo-historical information systems.

The model we propose in this paper addresses the evolution process of a single spatial entity and the multi-linear evolution of a set of spatial entities. The evolution of an independent spatial entity involves changes in its size and shape as well as movement processes. The multi-linear evolution represents changes of a set of spatial entities that are of the same spatial type and that covers the same area, the identified processes are the split of a spatial entity and the unification or re-allocation of several spatial entities. The framework is based on a conceptual representation of spatio-temporal processes defined in previous work (Claramunt and Thériault 1995, 1996). In this paper, we extend the former model semantics with a qualitative representation based on a mathematical description of spatio-temporal processes. This development aims at providing a sound model support (i.e. physically and logically possible spatio-temporal relationships). This spatio-temporal model may later contribute to support TGIS software development (e.g. spatio-temporal query languages and search algorithms). The resulting rules may be used to validate spatio-temporal relationships if they are explicitly stored in the database, or otherwise to compute and discover them.

The paper is organised as follows. We present a set of basic properties of space in Section 2 and a set of minimal topological relationships in Section 3. Properties of time are examined in Section 4. Section 5 introduces the notion of change and a set of temporal properties that link evolving entities. Section 6 describes the fundamental properties of evolving spatial entities. In Section 7 we develop a qualitative model for representing the linear evolution of a spatial entity. In Section 8 the multi-linear evolution of spatial entities is modelled. Section 9 concludes the paper and outlines future work.

2. Properties of space

Two types of phenomena can be represented in space. The first type is physically bounded and is usually assimilated, for scientific purposes, to a semi-closed system (e.g., a building or a car). There is little uncertainty about the limits of these real-world entities and their semantic definitions are generally universal and unambiguous. The second type is continuously spread over space with no obvious limits but transition areas between states measured on continuous scales. For purposes of analysis, scientists often define interpreted semantic classes that cover persisting regions of space sharing similar values (Newton-Smith 1980). The land they cover is delimited using a specific set of taxonomic rules that may change according to the application. Resulting territorial extensions are defined using geometric entities (e.g., soil categories, air masses, administrative regions). Even if it is not always straightforward to distinguish between real and interpreted entities, their distinction allows for the identification of particular properties: a real entity is represented in a topological space by a mapping function between a frame of reference and the topological space (e.g. a car or a building projected in a 2-dimensional space) while an interpreted entity is directly represented in the topological space.

The core of the proposed model is based on spatial entity primitives which describe real or interpreted entities that are discrete and located in space. A class of spatial entities represents a collection of spatial entities sharing similar generic characteristics. Each of these classes is defined by a type with its owns generic and specific attributes. A spatial attribute describes the geometry of a spatial entity. For the purpose of this paper, its value domain is restricted, to 2-dimensionnal (2-D) space, consisting of: points (0 dimension), lines (1 dimension, non closed without self-intersection) and polygons (2 dimensional contiguous patch of space with no hole). The properties of topological spaces have been extensively used as a mathematical support for GIS design. They provide a formal support for the expression of spatial relationships (Alexandroff 1961). Let us consider a 2-D topological space with a minimum set of properties: a spatial relationship model based on point set topology (Herrings 1991) and a metric based on Euclidean geometry. Topological operators are defined in topological spaces, informally:

- A spatial entity (e) is a subset of the topological space (X) that is described by a complement, a closure, an interior and a boundary.
- The complement of a set (X - e) is the collection of points that surrounds the set.
- The closure of a set (e^{-}) is the intersection of all the surrounded sets.

- The interior of a set (e°) is the collection of points completely surrounded by the set.
- The boundary of a set (∂e) is the set of points that intersects both the set and its complement. The boundary of a point feature (p) is empty ($\partial p = \emptyset$). The boundary of a non-connected line is the set of the two separate end-points.

3. Topological relationships in space

Spatial relationships in 2-D spaces are formally defined by topological operators (Egenhofer 1991, Cui 1993, Clementini 1993, Abdelmoty 1995). They generally differ in the type of spatial entities they cover (e.g., region to region, line to region relationships). Clementini proposes a generic model based on the topological intersections of two entities in space. This model is nearly independent of the geometric type (i.e. point, line or polygon) and presents the advantage of identifying a reduced number of relationships. Those topological relationships are summarised in Figure 1. A spatial entity (e or f) is defined as a 2-dimensional point-set (polygon, line or point) where its three components (interior, boundary and closure) are connected.

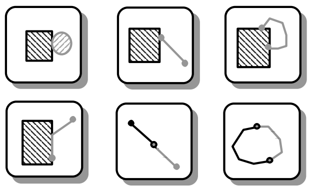
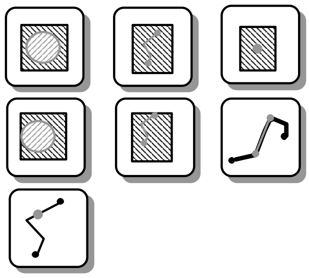
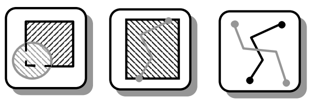
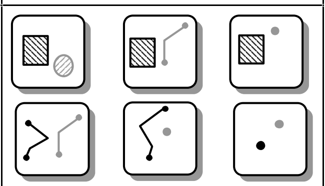
e equal f	$\Leftrightarrow$	$(e^\circ \cap f^\circ = e^\circ \cup f^\circ) \wedge (\partial e \cap \partial f = \partial e \cup \partial f)$	
e touch f	$\Leftrightarrow$	$(e^\circ \cap f^\circ = \emptyset) \wedge (\partial e \cap \partial f \neq \emptyset)$	
e in f	$\Leftrightarrow$	$(e \cap f = e) \wedge (e^\circ \cap f^\circ \neq \emptyset)$	
e cross f or e overlap f	$\Leftrightarrow$	$(e \cap f \neq e) \wedge (e \cap f \neq f) \wedge (e^\circ \cap f^\circ \neq \emptyset)$	
e disjoint f	$\Leftrightarrow$	$e \cap f = \emptyset$	

Figure 1 - Spatial topological relationships

We group the cross and overlap relationships since they only differ by the geometric dimension of the resulting intersection and this distinction is not significant for the purpose of this paper. The touch and cross/overlap relationships are not defined for points. With the exception of the 'in' operator, all the other relationships are symmetric. The 'in' and 'equal' relationships are transitive.

4. Basic properties of time

For the purpose of the following model, time defines a structure to label and link evolving spatial entities (things in time). This temporal structure is represented as a linear time-line which supports partial ordering. The characterisation of time is defined with metric properties (measured time leading to chronology) and topological properties (ordered events and interaction networks). A temporal metric assigns values to things in time and measures duration whereas a temporal topology allows the study of properties which are preserved under all continuous transformations.

Two time structures are generally used to represent a temporal property: point-oriented (instant) or interval-oriented logics (duration). Although our model uses an interval approach, it is also applicable, with minor adaptations, to an instant logic approach. Moreover, it is not dependent on the choice of a continuous or discrete time-line representation. A continuous time-line means that any interval of time is divisible since it is expressed with real numbers; whereas a discrete time-line means a finite representation where time is measured using integers. We use Allen's interval logic as it provides a complete set of temporal operators (Allen 1984). Temporal operators are defined through mutually exclusive relationships between time intervals (Figure 2). Begin(I) and End(I) are temporal operators which provide respectively the beginning and ending instants of a time interval I.

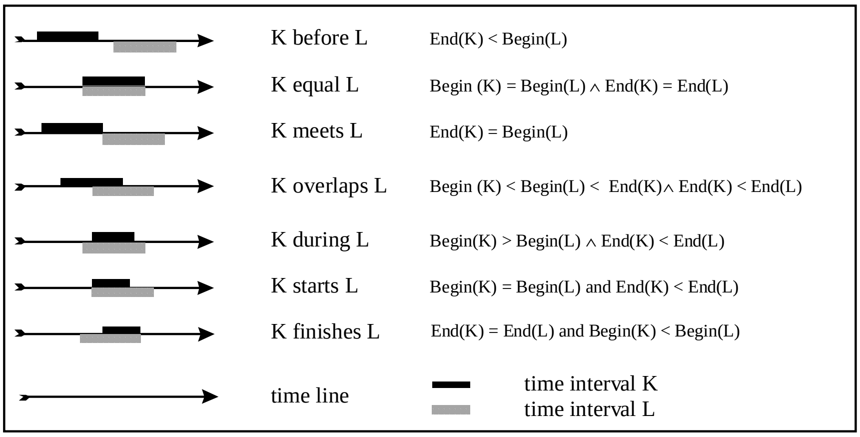


Figure 2 - Temporal relationships

Change happens when there is an alteration of any attribute describing a real-world entity or a geographic region (Newton-Smith 1980). However, spatio-temporal models must distinguish between trend and fluctuation changes (Miles 1992). A trend is an average change over a significant period of time leading from one state (the value of any attribute) to another (Bornkamm 1992). In structural terms, a trend changes the equilibrium point of the entity sub-system (i.e. entity attribute values) for a significant duration, without modifying the entity type (as opposed to the mutation process). Conversely, a fluctuation is a rapid change in attributes that is not stable within the application's time granularity, or that is related to observation bias. Fluctuations are reversible and oscillate around the average trend values. Therefore the proposed model will only consider trend variations as they lead to more meaningful changes.

5. From chronology to history

To give a more precise signification of change, we define the state of an entity:

- The state of a spatial entity e, denoted as e_i, represents a stable value (without trend change) of this entity within an interval of time I_i as (Allen 1984): $Holds(e_i, I_i) \Leftrightarrow \forall\, t \in I_i, Holds\_at(e_i, t)$;
- An entity state holds throughout every sub-interval included in its own temporal interval: $Holds(e_i, I_i) \wedge J \subseteq I_i \Rightarrow Holds(e_i, J)$.

We characterise the evolution of an entity (e) with its two successive linear states (e_i and e_{i+1}) defined respectively within I_i and I_{i+1} intervals of time. In order to represent the linear structure of time, these intervals are constrained:

- $Holds(e_i, I_i) \wedge Holds\ (e_{i+1}, I_{i+1}) \Rightarrow [End(I_i) \le Begin(I_{i+1})]$.

Evolving spatial entities use an interaction network to link their two successive (e_i, ..., h_m) and (e_{i+1}, ..., h_{m+1}) state sets defined respectively within (I_i, ..., I_m) and (I_{i+1}, ..., I_{m+1}) intervals of time (Figure 3) . Their respective time intervals are also constrained:

- $[Holds(e_i, I_i), ..., Holds(h_m, I_m)] \wedge [Holds(e_{i+1}, I_{i+1}), ..., Holds(h_{m+1}, I_{m+1})]$
$\Rightarrow [Maximum(End(I_i), ..., End(I_m)) \le Minimum(Begin(I_{i+1}), ..., Begin(I_{m+1}))]$.

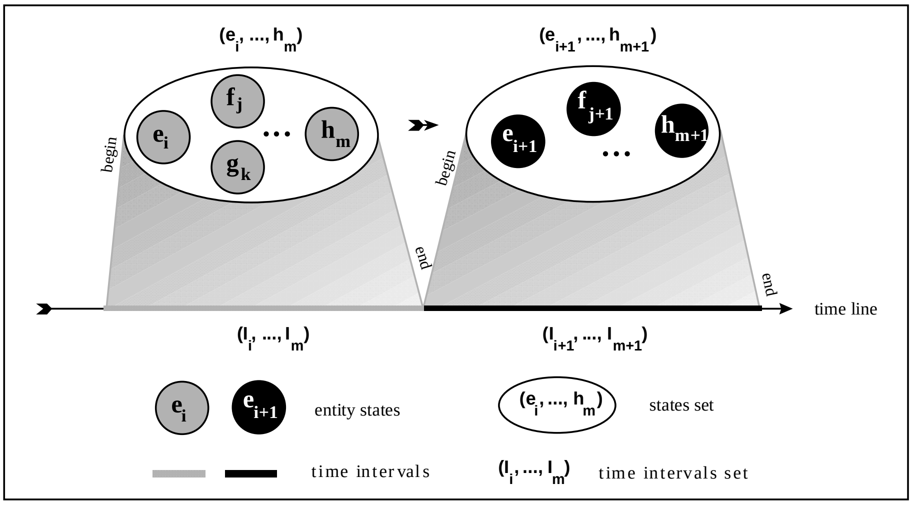


Figure 3 - Multi-linear evolution of spatial entities

Linear and multi-linear evolving entities share temporal properties (Figure 4):

- e_i and e_{i+1} are two immediate states of an entity if and only if $End(I_i) = Begin(I_{i+1})$;
- e_i and e_{i+1} are two delayed states of an entity if and only if $End(I_i) < Begin(I_{i+1})$;
- (e_i, ..., h_m) and (e_{i+1}, ..., h_{m+1}) are two immediate sets of entity states if and only if $End(e_i) = ... = End(h_m) = Begin(e_{i+1}) = ... = Begin(h_{m+1})$;
- (e_i, ..., h_m) and (e_{i+1}, ..., h_{m+1}) are two delayed sets of entity states if and only if $End(e_i) = ... = End(h_m) = Begin(e_{i+1}) = ... = Begin(h_{m+1})$ is false;

- Delayed and immediate states represent two possible time-ordering relationships between two non overlapping sets of time intervals.

The represented world is discrete in both space and time. Discontinuity in the life of an entity is assumed in the time line and represented by delayed entity states. This property is required in order to represent episodic phenomena (e.g., successive recorded positions of a moving spatial entity, weekly measurement of the water quality at a beach station). There is an analogy between this time discontinuity and the discrete structure of space used in physics (e.g., in the Bohr-Rutherford model where the position of an electron changes in a discrete quantized manner).

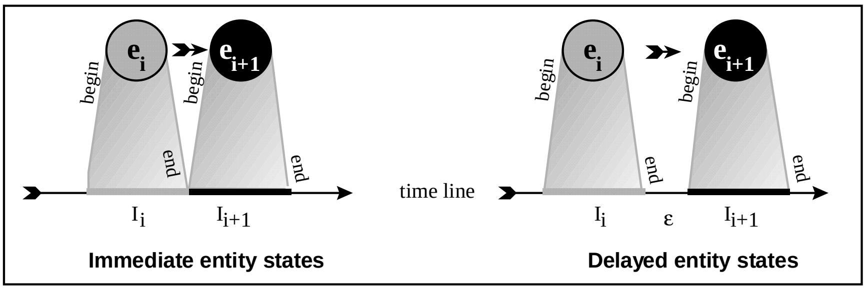


Figure 4 - Successive entity states

6. Space and time properties

A space and time model completes the static properties of a spatial entity by providing dynamic characteristics and integrating the following properties for an evolving spatial entity:

- the state of a spatial entity, denoted (e_i), is characterised by its location in space and by its form, defined by the union of its interior and its boundary ($e_i^\circ \cup \partial e_i$);
- the location of a spatial entity state, denoted $P(e_i)$, is defined by a projection of its centre of gravity into a 2-D Euclidean coordinate system ($\Re^2$) $\Rightarrow$

  $P: X \rightarrow \Re^2$ and $e_i \rightarrow \{x(e_i), y(e_i)\}$.

Two states of two distinct spatial entities e_i and e_j share the same location in space (i.e. $P(e_i) = P(e_j)$) if and only if $[x(e_i) = x(e_j)] \wedge [y(e_i) = y(e_j)]$. Galton defines the position of an entity as the total region of space it occupies at a given time (Galton 1995). We go further and represent the total region of space occupied by a geographic entity as the union of its interior and boundary plus the location of its centre of gravity. The distinction can thus be made between contraction, expansion and deformation from translation and rotation evolution through analysis of the locational change (centre of gravity displacement) and the topological intersection between two successive states of an entity. Describing change for GIS leads to the definition of the spatial evolution of geographic entities. A spatial evolution represents a trend change over space. It is characterised by the modification of the values of its spatial properties (geometrical and topological components). The model integrates two kinds of evolving spatial entities:

- Those that are free to change, and whose semantics are defined from an application point of view without geometrical or topological constraints (e.g. a vehicle, a person, an hurricane). They can be modelled using a single entity approach and the usual time line.

- Those whose changes are constrained by topological, positional or structural relationships with other entities (e.g., within a cadastral database, lots must remain adjacent and cover the entire county at any time, even when they exchange their territory). They need a network time model to express their joint evolution and to retain these complementary relationships using the multi-linear evolution approach.

The distinction between independent and related entities must be fixed in accordance with the application requirements because the same entity type (e.g., vehicles) may be seen as free to move for some purpose (e.g., a ship sailing over the sea), or restricted by the geometry and topology of a transportation network for another application (e.g., a car moving within a city's street network).

7. Linear evolution of an independent spatial entity

This section presents the basic spatio-temporal relationships defined for an independent entity using the concept of linear evolution. Two successive immediate or delayed states (e_i and e_{i+1}) of a single spatial entity represent an isomorphic evolution if they are respectively members of two isomorphic topological spaces X_1 and X_2. This happens if and only if there exists an isomorphism H (i.e., a mapping function which maintains the shape and size of spatial entities) between the X_1 and X_2 topological spaces. Considering the positional and the morphologic properties of two successive spatial entity states (location, shape, size and orientation) we obtain the following five minimal and orthogonal cases (Figure 5):

- stability is provided to express the invariance of spatial components;
- expansion and contraction are inverse forms of homeomorphism i.e. a mapping function that preserve shape and orientation (i.e. size change, location may change);
- deformation is an evolution that implies possible simultaneous change in size and orientation (i.e. shape change, size and location may change);
- translation involves movement in space while maintaining shape and size (isomorphism), and orientation (i.e. location change);
- rotation implies locational stability and isomorphism but change of orientation.

Isomorphic spatio-temporal relationships (rotation and translation) have an equivalence property: the topological relationships between e_i and e_j in X1 are said to be equivalent to the topological relationships between e_{i+1} and e_{j+1} in X2 if and only if there is an isomorphism between X1 and X2. Non isomorphic transformations are restricted to possible relationships between successive states, three cases are identified: expansion, contraction and deformation. Other combinations are not valid according to the following restrictions:

- cross and overlap are commutative relationships: e_i cross/overlap e_{i+1} is equivalent to e_{i+1} cross/overlap e_i;
- touch and disjoint relationships are not used since they imply a de facto translation;

- topological equality is irrelevant as well because it is not compatible with the non isomorphic property.

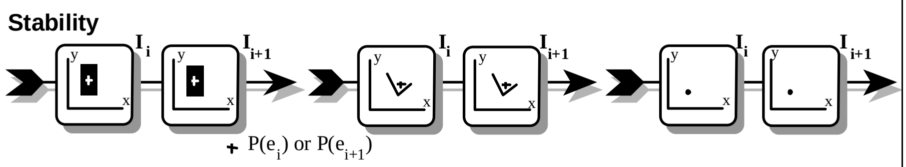
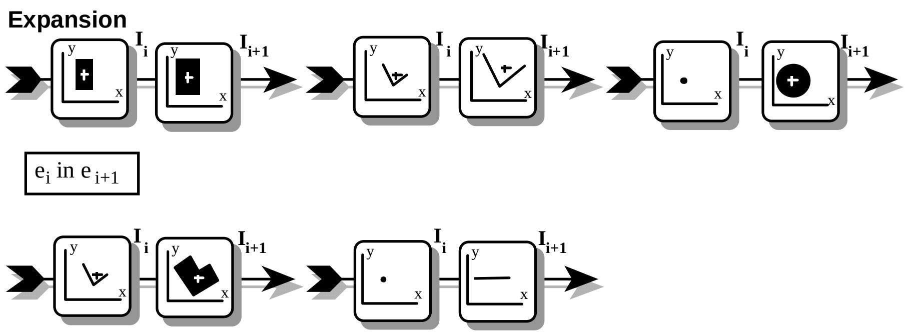
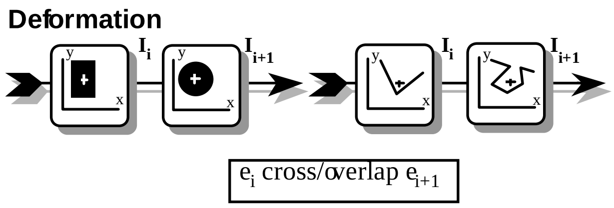
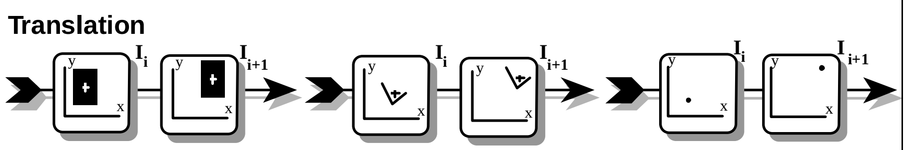
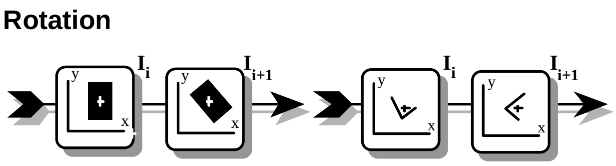
• $[e_i^\circ \cup \partial e_i] = [e_{i+1}^\circ \cup \partial e_{i+1}]$ • $P(e_i) = P(e_{i+1})$ • $End(I_i) \leq Begin(I_{i+1})$	
• There is no H / $H(e_i^\circ \cup \partial e_i) = [e_{i+1}^\circ \cup \partial e_{i+1}]$ • $P(e_i) = P(e_{i+1})$ or $P(e_i) \neq P(e_{i+1})$ • $End(I_i) \leq Begin(I_{i+1})$	 **Contraction** is the inverse expansion relationship (with e_{i+1} in e_i)
• There is no H / $H(e_i^\circ \cup \partial e_i) = [e_{i+1}^\circ \cup \partial e_{i+1}]$ • $P(e_i) = P(e_{i+1})$ or $P(e_i) \neq P(e_{i+1})$ • $End(I_i) \leq Begin(I_{i+1})$	
• $\exists H / H(e_i^\circ \cup \partial e_i) = [e_{i+1}^\circ \cup \partial e_{i+1}]$ • $P(e_i) \neq P(e_{i+1})$ • $End(I_i) \leq Begin(I_{i+1})$	
• $\exists H / H(e_i^\circ \cup \partial e_i) = [e_{i+1}^\circ \cup \partial e_{i+1}]$ • $[e_i^\circ \cup \partial e_i] \neq [e_{i+1}^\circ \cup \partial e_{i+1}]$ • $P(e_i) = P(e_{i+1})$ • $End(I_i) \leq Begin(I_{i+1})$	

Figure 5 - Spatio-temporal relationships
for an evolving spatial entity

Particular restrictions apply for point evolution:

- point deformation and rotation are irrelevant since a point is dimensionless and has no shape or orientation;
- point contraction is equivalent to disappearance;
- point expansion implies a geometric type mutation from point to line or from point to polygon.

Combining these five basic operators is sufficient to model complex movements of an independent entity in a 2-D Euclidean space. However, we need networks to describe complex movements of droves grouping many independent entities (Cheylan 1993) or to represent co-ordinated evolution of related spatial entities (Frank 1994, Claramunt and Thériault 1996). We discuss the multi-linear evolution of spatial entities in the next section.

8. Multi-linear evolution of related spatial entities

As the range of phenomena that can be processed in a temporal GIS is probably inexhaustible, modelling all possible cases of multi-linear evolution involving related spatial entities is a complex task and one far beyond the scope of this paper. This section analyses the redistribution of geometric space among a set of geographic entities of the same spatial type that collectively define an exhaustive coverage and exclusive partition of a subset of the topological space. Since multi-linear relationships often imply sets of evolving spatial entities, we introduce a set of symbols to describe their characteristics and operations:

- $\{E_1\} = (e_i, f_j, ..., h_m)$ and $\{E_2\} = (e_{i+1}, f_{j+1},..., g_k)$ are two consecutive (immediate or delayed) sets of spatial entities;
- $\{E_1\}°$ is the union of interiors of $e_i, f_j, ..., h_m$;
- $\partial\{E_1\}$ is the union of boundaries of $e_i, f_j, ..., h_m$;
- $\{E_1\}° \cap \{E_2\}°$ is the intersection of the interiors of $(e_i, f_j, ..., h_m)$ with those of $e_{i+1}, f_{j+1}, ..., g_k$;
- $\partial\{E_1\} \cap \partial\{E_2\}$ is the intersection of boundaries of $(e_i, f_j, ..., h_m)$ with those of $e_{i+1}, f_{j+1}, ..., g_k$.

A redistribution of space involves a set of spatial entities and their topological components (Figure 6). They together make an exhaustive coverage of an overall territory (e.g., counties in a country) but they do not overlap (i.e., exclusive partition of space). This generic class of multi-linear evolution has applications in various fields: property management, political division of land to avoid jurisdictional conflicts, street names management to provide unique addresses. They are also useful for building diachronic series with census data, to avoid the effects of an eventual rezoning of statistical areas.

Figure 6 illustrates three basic forms of geometric redistribution used to restructure a network of connected lines or adjacent polygons: split, union and re-allocation. The multi-linear evolution adds an additional dimension to these networks since they must also be connected in time. All these spatio-temporal relationships share the same basic requirements:

- successive entity sets must have the same geometric type;
- they must exist during their specific time intervals and they must use line or polygon primitives to display their position in space;
- they must be connected in time (immediate or delayed successors);
- they entirely cover the same geometric extension (exhaustivity) without overlapping (exclusivity).

Successive entity sets are constituted with mutually exclusive geometric primitives that are connected in space (connected lines or adjacent polygons) before and after the change occurs. There is some change in the internal boundaries layout. The only factor that differentiate them is the composition of the previous and subsequent entity sets; they are one-to-many (split), many-to-one (union) or many-to-many (re-allocation) relationships. The later is more difficult to implement because it implies an algorithm or an heuristic to find the minimum previous set of entities ($\{E_1\}$) that perfectly overlaps ($\{E_1\} \cap \{E_2\} = \{E_1\} \cup \{E_2\}$) a corresponding minimum subsequent set of entities ($\{E_2\}$).

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- $[e_i^\circ \cup \partial e_i] = [\{E_1\}^\circ \cup \partial\{E_1\}]$
- $[\partial\{E_1\} \cap \partial e_i] \neq [\partial\{E_1\} \cup \partial e_i]$

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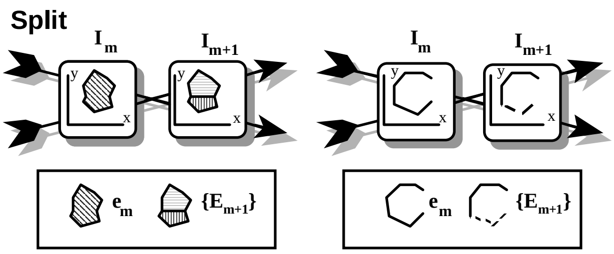

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- $[\{E_m\}^\circ \cup \partial\{E_m\}] = [\{E_{m+1}\}^\circ \cup \partial\{E_{m+1}\}]$
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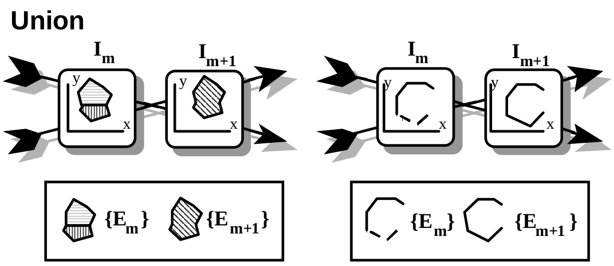

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- $[\{E_m\}^\circ \cup \partial\{E_m\}] = [\{E_{m+1}\}^\circ \cup \partial\{E_{m+1}\}]$
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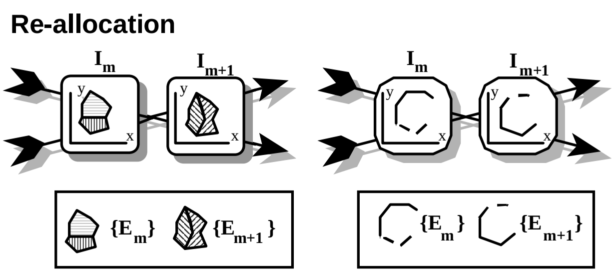

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Figure 6 - Redistribution of polygons and line segments between related geographic entities

9. Conclusion

The development of temporal GIS needs formal models to represent relationships in both space and time. Qualitative reasoning, which is widely used to define spatial relationships, must therefore be extended to the temporal dimension to provide a set of well-defined, homogeneous spatio-temporal relationships.

This paper introduces a general method to represent qualitative relationships for evolving spatial entities. The model allows the formalization of the evolution of spatial entities. It is flexible and applies to 2-dimensional spatial entity types. We identify a set of minimal spatial evolutions for a single spatial entity (stability, expansion/contraction, deformation, translation and rotation) and a particular case of multi-linear spatial evolutions (split, union and re-allocation). Further work concerns the model extension toward the representation of functional relationships such as replacement and diffusion processes.

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